# FREQUENT FLYER PROGRAMS, MORAL HAZARD AND REWARDS PER MILES VS PER DOLLAR[1]

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**ABSTRACT**

Since their inception in 1981, airlines' frequent flyer programs have become one of the largest customer loyalty programs in the world economy. The classic and oldest approach used is to assign rewards based on the distance traveled, but recently many Low Cost carriers (e.g. Southwest, 2011) and Legacy carriers (e.g. Delta, 2015) have moved to a rewards-per-dollar paid approach. We propose a theoretical model that considers the existence of two types of travelers, business (who do not pay for their ticket) and leisure, so that the (monopoly) airline face an adverse selection and a moral hazard problem. We use this model to study the differences between a rewards-per-distance program versus a rewards-per-dollar program, using an 'optimal' program as a benchmark.
*Keywords: Frequent-flyer program, adverse selection, moral hazard.*

## 1. INTRODUCTION

Since their inception in the 80s, airlines' frequent flyer programs (FFP) have become one of the largest customer loyalty programs in the world economy. The aeronautical industry places unredeemed miles in over 700,000 million dollars. However, despite their size and importance, these programs have remained relatively unexplored from a microeconomics point of view. Caminal & Claici (2007), Hartman & Viard (2008), Lederman (2007), Agostini et al (2013) have focused on the pro- or anti-competitive effects that FFPs might have, emphasizing the potential switching costs that they would create, namely, the benefit losses suffer by consumers that switch to another airline.

An alternative analysis of FFP's is provided by Basso, Clements & Ross (2009) who observe that, in many cases, it is not the traveler who pays his ticket but a third-part payer. This is the main case in most business-related travels, where the employer is the one who pays. Their analysis emphasizes the fact that the employee might not always choose the cheapest ticket but will try to maximize individual benefits and it is there where FFP rewards play a major role: the miles or rewards that airlines offer, which are received by the traveler, might be seen as a sort of bait –or bribe– to attract travelers towards higher price alternatives. We therefore have a moral hazard

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problem: employers, who pay for air tickets are not able to observe the actions –in the case, choice of airline– of their employees but, in many cases, can just only accept or reject the proposal. Basso et al (2009) show that in such an environment, a lone airline can indeed take advantage of the third-party payer feature, increasing prices up to the employers' reservation price, beating the competition. Yet, if all airlines use FFPs, rewards become simply a new competitive instrument which enables tougher competition: airlines will dissipate profits through FFP rewards in a standard Prisoner's Dilemma fashion. Prices will remain high though, showing that all benefits flow from employers and airlines to employees.

Basso et al (2009) suppose, however, that airlines are perfectly able to discriminate business from leisure travelers –who do pay for their tickets– and are, therefore, able to increase prices and miles to the former, without losing demand from the latter. In the economics literature, this is known as first degree price discrimination. However, firms in general and airlines in particular can't easily distinguish one type of traveler from another, which implies that airlines face, also, an adverse selection problem, something actually well documented in the business literature.

Under these circumstances, the airline can simply avoid attempting to separate the travelers, in which case the firm will face the combined demand, choosing one price and one level of rewards (if any) for all customers. A second option is to offer different menus or pair of prices and rewards, attempting to induce self-selection by travelers into the menu designed for them. In this case then, the bribe is not designed to attract consumers away from smaller prices of competitors but of the airline's own small prices. This case might seem the one that prevails. Air Canada, for example, has a "Tango" fare which, together with other restrictions, only gives 25% of the miles. The "Latitud" fare, however, while being larger, gives 125% of the miles. LATAM airlines offer something similar while Southwest, a low cost carrier, has also three classes, but in their case, the rewards are linked to the price paid: higher prices obtain a reward that is 12 times the price paid, the intermediate class obtains 9 times the price paid, while the lowest prices obtains only 5 times the (lower) price paid. Other low cost carriers have also Southwest's system (JetBlue and Virgin America).

It is evident that the problem explained above is related to the second-degree price discrimination literature –in many cases, it has been used as a textbook example– where the firm offer heterogeneous consumers a well-crafted menu of options, with the intention to induce them to self-select and thus maximize profits. This literature is ample, starting with the seminal work by Mussa & Rosen (1978) and is well reviewed in Stole (2007). Our contribution is to consider simultaneously the moral hazard problem between employer and employee regarding the choice of airline and the adverse selection problem that airline faces.

Thus, in this paper we propose a theoretical model that considers the existence of two types of travelers, business and leisure, who are not distinguishable by the (monopoly) airline ex-ante, and which is the source of the adverse selection problem. We also consider that business travelers do not pay their full fare, inducing a moral hazard problem between them and their employers. The airline then faces a second-degree price discrimination problem, where it attempts to separate traveler types with the use of different menus of prices and rewards, while exploiting the moral hazard problem by 'bribing' business travelers. We look at the interplay of these two aspects of the airline problem, seeking to understand the trade-offs and the effects on the market (prices and rewards), efficiency and distribution of surpluses of all different agents involved (business travelers, leisure travelers, employers and airlines). We focus on the classical framework of second degree price discrimination where the demand is of the all or nothing type; relaxing that assumption allowing elastic demands does not change the qualitative conclusions.

Our main results show that in the case of the full information benchmark, the airline finds it optimal to provide both types with positive rewards but in the case of the business traveler, this amount is inefficiently high, that is, airline's marginal cost exceeds traveler's marginal benefit. This is caused by the effect of the third-party payer, which makes a business traveler less elastic with respect to price. When information is incomplete, the reward to the business traveler remains the same yet those assigned to a leisure traveler decreases. This effect is known as *no distortion at the top*, and happens because the airline needs to separate enough the two menus to induce self-selection by customers. In fact, the airline might find optimal not to offer any reward to leisure travelers but charge a low price, while having a different option with very high prices and rewards. Business prices might end up being too high for what employers are willing to pay though; to consider this we add the employer's reservation price, which establishes a ceiling for airfares. This reservation price will induce a decrease in the rewards received by employees and an increase in the rewards received by leisure travelers, moving both rewards levels closer to their efficient levels.

This first analysis, however, may be understood as a point of reference: the airline is free to set prices and rewards yet, in practice, real programs link their rewards, in an explicit contract with their customers, to some other variable. Indeed, both Air Canada and LATAM link the rewards to the distance flown (hence, the old idea of obtaining 'miles') yet Southwest links the rewards to the price paid. The analysis of real FFPs and how they compare between them, and against what we will call the 'optimal program' is the subject of the second part of the paper. The importance of this analysis is further fueled by a couple of facts from the industry: first, in 2015 Delta followed the lead of Southwest and moved its Skymiles program to a per-dollar system.[2] United then followed the lead of Delta[3] and, very recently, LATAM announced that for local flights, rewards would be also linked to fares. [4] Is there, then, an advantage to using rewards linked to prices rather than distance flown? The second observation relates to class shutdowns. We collected data from three airlines, namely LATAM Airlines, Air Canada and Southwest Airlines, from October and November of 2014, referring to one-way travels for four pairs of cities in Chile, Canada and the U.S.[5] At the time of the data collection, LATAM Airlines and Air Canada offered a rewards-per-distance program, while Southwest used a rewards-per-dollar program. The data showed that the airlines effectively offered at least two fares with different rewards (three fares in the case of Air Canada and Southwest and four in the case of LATAM). Moreover, this data also revealed that LATAM and Air Canada often closed the lower classes: LATAM closed the fourth, third and second tier class 68%, 53% and 26% of the time respectively, while Air Canada closed the third and second tier class 34% and 4% of the time respectively. On the other hand, the data for Southwest showed that the lowest tier class was closed only once (<1% of the time). Is there

[2] See e.g. https://www.usatoday.com/story/travel/flights/2014/02/26/delta-frequent-flier/5815425/.

[3] See e.g. https://www.usatoday.com/story/todayinthesky/2014/06/10/united-fliers-to-earn-miles-based-on-fare-not-distance/10270819/

[4] See e.g. https://www.latam.com/es_cl/laser_latam_pass/mailv/juntosmaslejosvoladores/

[5] Chicago-Boston (Southwest), Toronto-Vancouver (Air Canada), Santiago-Puerto Montt and Santiago-Antofagasta (LATAM).

something structural about the rewards-per-dollar program that makes it more unlikely to shut down classes?

To study the real FFP programs we consider two settings. First, in the ‘ex-ante’ setting, airlines design their programs for expected parameter values. A program design means fares and rewards in the optimal program, fares and percentage of miles in the per-distance program, and fares and fare-multipliers in the case of the per-dollar program. We show that in this case all three programs are equivalent, in terms of fares and resulting rewards. This is irrespective of whether separation (as in the standard adverse selection result) or shutting down the lowest class is better: ex-ante, both real programs can mimic the optimal program.

The second setting we analyze is the ‘ex-post’ setting. What we mean here is that after designing the program, demand parameter values may differ from their expected values.[6] In the optimal program, the airline would be able to adjust both rewards and fares yet in real programs, we assume the airline is tied to the ex-ante program design, that is, the airline cannot change the percentages in the per-distance program or the fare-multipliers in the per-dollar program. Thus, in the ex-post setting an airline can imperfectly adjust values of fares and reward, and the way this happens do depend on the type of program.

Our analysis of the ex-post setting shows that in the per-distance program, rewards cannot be changed, as they are tied to the distance, while in most cases the airline would like to change all rewards and fares. The sub optimal solution is, in many cases, to move fares in the opposite direction of what the airline would do if it had total freedom. The per-dollar program has rewards tied to prices, and therefore, both fares and rewards move, although at a fix rate. This characteristic, while being suboptimal, can be shown to, at least and under some conditions, allow the airline to move fares and rewards in the same direction as if it had total freedom. In other words, in the ex-post setting, the per-price program gives allows the airline to adjust better. This robustness of the per-dollar program may explain why it is being adopted by legacy carriers. Furthermore, we show that the condition for profitable class shutdown differs between programs and that, indeed, it is more easily reached in the per-distance program, i.e. our models predict that shutting down the lower classes is less likely with a per-dollar program, just as the data we collected shows.

The rest of the paper is organized as follows: Section 2 contains the model formulation and the benchmark case of full information, that is, perfect discrimination. Section 3 considers the adverse selection and moral hazard problems, and characterizes what we have called ‘the optimal program’. Section 4 considers the formulation and analyses of real programs, namely rewards-per-distance and rewards-per-dollar, in the ex-ante setting. Section 5 show the analysis for real programs in the ex-post setting, with comparisons between programs and against the optimal program, including shutdown conditions. Section 7 concludes.

## 2. MODEL FORMULATION AND PERFECT DISCRIMINATION BENCHMARK

[6] According to the marketing literature, a customer loyalty program, to be successful, need to be, among other things, simple and stable over time. Hence, the design of the program is very slow to adjust.

Our model considers two types of travelers, business and leisure, who make the purchase arrangements by themselves, and an airline that possess a frequent flyer program and that has all customers enrolled in. Miles or rewards flow to the traveler, and not to the payer of the ticket. The business traveler only pays, or perceives a fraction of the air ticket and is forced to travel as an employment condition. This 'fraction' attempts to capture that, while not paying, the employer is partially sensitive to the price, either because he may care about future travel, and therefore needs to be careful about the travel account, or because he fears he will be audited. The leisure traveler pays the ticket from his own pocket and flies if this leaves him with positive utility.

Travelers utility functions are represented by an additive and separable function of price and reward benefits similar to the one used by Rochet & Stole (1997, 2002). To model the existence of business travel we follow Cairns & Galbraith (1990) and Basso et al (2009) where these types of consumers only pay a small fraction α of the airfare. Therefore, utility functions are described by:

$$U_L(P_L, F_L, z) = U_0 + \theta_L V(F_L) - P_L$$

$$U_H(P_H, F_H) = \theta_H V(F_H) - \alpha P_H$$

With $P_i$ the price charged and $F_i$ the level of rewards, where L refers to leisure travel and H to business travel (*Low* type and *High* type), and $U_0 > 0$, $\theta_L \leq \theta_H$, $0 \leq \alpha \leq 1$, $0 < V(0)$, $V'(F) \geq 0$ y $V''(F) < 0$. Total market demand is $N_H + N_L$, known by the airline, where $N_H$ is the number of passengers of *High* type and $N_L$ the corresponding of *Low* type. Note that the business traveler has no personal utility of travelling, this is because travelling is a condition of employment. On the other hand, the travel for the leisure passengers can be seen as a way to access some other leisure activities, and so it has intrinsic value for them. Mokhtarian & Salomon (2001) comment about the concept of leisure travel as a derived demand when the increased attractiveness of the destination outweighs the disutility of travel required to reach it.

We set airline's operational marginal cost to zero and suppose that its reward cost structure is separable, that is:

$$C(F_H; F_L) = C_L(F_L) + C_H(F_H)$$

Rewards marginal costs are increasing and convex, i.e. $C_L{}'(F) > 0$ , $C_H{}'(F) > 0$ and $C_L{}''(F) > 0$.

The airline will generate a menu with two pairs of prices and rewards, each targeted at each type of traveler. These pairs, to induce truthful self-selection must be designed such that they fulfill incentive compatibility constraints:

$$U_L(P_L, F_L, z) \geq U_L(P_H, F_H, z) \tag{1}$$

$$U_H(P_H, F_H) \geq U_H(P_L, F_L) \tag{2}$$

The monopoly airline must also ensure participation of customers. For leisure traveler, this simply requires that utility end up being positive. In the case of the business traveler, given that traveling is a condition for employment, his participation constraint is not her own but the employer's, which is modeled through a reservation price $R$, which is the maximum price that she is willing to pay for an air ticket. Participation constraints are, then:

$$U_L(P_L, F_L, z) \geq 0 \tag{3}$$

$$P_H - R \geq 0 \tag{4}$$

Note that we are modelling all-or-nothing type of demands, as in the most common adverse-selection model: either all business (leisure) traveler fly, or none does. Introducing elastic demands doesn't change the main qualitative results and insights.

We now establish the airline's optimal decisions for the full information benchmark, that is, a case in which the airline can perfectly discriminate between customers. The prices and rewards chosen are then described by:

$$\theta_L V'(F_L^*) = C_L{}'(F_L^*) \tag{5}$$

$$\frac{\theta_H}{\alpha} V'(F_H^*) = P_H{}^* = R \tag{6}$$

$$P_L{}^* = \theta_L V(F_L^*) \tag{7}$$

$$P_H{}^* = R \tag{8}$$

In this case the airline has an incentive to keep increasing the price for business travelers, up to the reservation price of the employer, while inducing self-selection through inflated rewards, which is shown by the fact that the marginal benefit for business travelers in (5) is divided by α.

## 3. ADVERSE SELECTION

We consider now imperfect information to include the adverse selection problem. To build intuition we assume first that the employer's reservation price is "very large" $(R = \infty)$, so that, for the time being, the airline does not consider their participation constraint in its profit maximization problem. The problem the airline faces is

$$Max_{(P_H, P_L, F_H, F_L)}\ \pi = N_L\big(P_L - C_L(F_L)\big) + N_H\big(P_H - C_H(F_H)\big)$$

s. t.

$$\theta_H V(F_H) - \alpha P_H \geq \theta_H V(F_L) - \alpha P_L$$
$$\theta_L V(F_L) - P_L \geq \theta_L V(F_H) - P_H$$
$$U_0 + \theta_L V(F_L) - P_L \geq 0$$
$$F_L, F_H \geq 0$$

At this point, it becomes very useful for solving the problem, to perform a change of variables (Rochet y Stole, 2002):

$$u_L = U_0 + \theta_L V(F_L) - P_L,\ \ u_H = \frac{\theta_H}{\alpha} V(F_H) - P_H \tag{9}$$

This change of variables can be interpreted as the firm choosing the level of rewards, given by $F_H$ y $F_L$, and a utility level for each customer, given by $u_H$ y $u_H$. With this, we obtain the rules that the airline follows in its optimal policy. Calculations are presented in the Appendix A[7], but as is usual, the incentive compatibility constrain that is active is that of the high type (business traveler), while the participation constraint that is active is that of the low type (leisure traveler). The other two are slack. The *optimal program* is then characterized by:

$$u_L^* = 0 \tag{10}$$

[7] Appendix available at Google Drive - Appendix.

$$u_H^* = V(F_L^*)\left(\frac{\theta_H}{\alpha} - \theta_L\right) - U_0 \tag{11}$$

$$\frac{\theta_H}{\alpha} V'(F_H^*) = C'_H(F_H^*) \tag{12}$$

$$\theta_L V'(F_L^*) = \frac{C_L'(F_L^*) - B}{1 - \frac{N_H}{N_L}\frac{\left(\frac{\theta_H}{\alpha} - \theta_L\right)}{\theta_L}} \tag{13}$$

Where B is the Lagrange multiplier associated with the non-negativity of $F_L$. Note that here business travelers obtain positive utility (equation 11)[8], as opposed to what would happen under perfect discrimination. The airline must leave this type of customers with informational rents to induce them to truthfully self-select. Regarding rewards, the upward distortion of business rewards still occurs (equation 12), just as in the case with full information and moral hazard. This inefficiency, therefore, occurs only due to the moral hazard problem induced by the third-party payer with imperfect monitoring. On the other hand, comparing equations (13) and (5) one can see that the allocation of rewards to leisure travelers does change; they diminish in this case. The intuition is simple: since now the airline must achieve customer self-selection, as it faces the adverse selection problem, it decreases the amount of rewards to the lowest type to make that option less attractive to business customers. In a nutshell, there will be now two distortions on the rewards side: upwards in the case of business travel, caused by the fact that they do not fully pay their fare, and downwards in the case of leisure travel to induce a larger difference (in terms of rewards) between the two options. This last effect, caused by adverse selection is well-known, but here we have an added effect caused by the moral hazard problem since $F_L$ now depends on α. This behavior of $F_L^*$ with respect to α can be seen graphically by considering the function

$$N(\alpha, F) = \frac{C_L'(F)}{\theta_L\left(1 - \frac{N_H}{N_L}\frac{\left(\frac{\theta_H}{\alpha} - \theta_L\right)}{\theta_L}\right)}$$

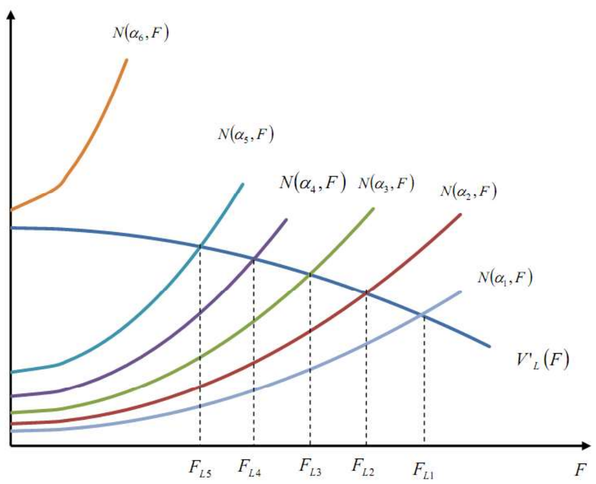


**Figure 1**. Variation of $F_L^*$ with respect to α ($\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4 > \alpha_5 > \alpha_6$).

[8] We assume $\alpha$ small enough such that $U_0 < V(F_L^*)\left(\frac{\theta_H}{\alpha} - \theta_L\right)$. If this is not the case, the business travelers' utility could in fact be dissipated. In any case, the qualitative results don't change.

From Figure 1 one can observe that, as $\alpha$ diminishes, the optimal value of $F_L$ also does: the third-party payer feature increases the differences in rewards. Now, if $\alpha$ is so low that the curves $V'(F)$ and $N(\alpha, F)$ no longer intersect, then the optimal airline's solution reaches a corner and then $F_L = 0$ (as it will be captured by B). It follows that there exist a critical $\alpha^*$ such that, ceteris paribus, $F_L^* = 0$ if $\alpha$ is smaller than this value. Zero reward for leisure travel causes no problem for the solution of the rest of the problem though, since $V(0)$ y $V'(0)$ are positive.

We can now look at prices. The analysis for $P_H$ shows that as $\alpha$ decreases $P_H$ will rise: the moral hazard problem induces higher prices. Leisure travelers, on the other hand end up paying the price that allows the airline to fully extract their surplus, i.e. $P_L = \theta_L V(F_L) + U_0$. Thus, as $\alpha$ diminishes, $P_L$ diminishes and so does $F_L$. Note that these results help explain the observed behavior of different alternatives with high price and a large amount of rewards (100% or 'more' of the usual miles number) or low prices with small rewards (such as 25% of the usual miles number).

Now, the results described up to here are valid only when it is desirable for the airline to accommodate both types. But, if for given parameter values the following condition is satisfied

$$N_L\left(P_L^* - C_L(F_L^*)\right) + N_H\left(P_H^* - C_H(F_H^*)\right) < N_H(P_H^{**} - C_H(F_H^{**})) \tag{14}$$

where $P_H^{**}$ and $F_H^{**}$ are the fare and reward offered to business passengers in the absence of leisure passengers, then the low class is closed and only the business passengers are served.

From the equations above, tedious yet straightforward calculations mapped in Appendix B[9] allow us to solve for comparative static results. These derivatives are summarized in the following table:

**Table 1**. Comparative statics for the optimal program.

| $\frac{d}{dx}$ | $F_H$ | $F_L$ | $P_H$ | $P_L$ |
|---|---|---|---|---|
| $\alpha$ | $\leq 0$ | $\geq 0$ | $\leq 0$ | $\geq 0$ |
| $\theta_H$ | $\geq 0$ | $\leq 0$ | $\geq 0$ | $\leq 0$ |
| $\theta_L$ | $= 0$ | $\geq 0$ | Depends | $\geq 0$ |
| $N_H$ | $= 0$ | $\leq 0$ | $\geq 0$ | $\leq 0$ |
| $N_L$ | $= 0$ | $\geq 0$ | $\leq 0$ | $\geq 0$ |

These results will be helpful in the ex-post comparison, as they show how fares and rewards should change if the ex-ante expected values are, ex-post, different. Note that, in all cases, at least one of the rewards is adjusted.

[9] Appendix available at Google Drive - Appendix.

## 4. REAL FFPs EX ANTE

### 4.1 Rewards per Distance

We may now analyze the performance of the programs observed in reality. We study the ex-ante setting, where the airline design the FFP program for the expected parameter values. We start by the classic and oldest per-distance program, where the rewards (miles) are based on distance traveled, that is

$$F_L = k_L \cdot D \qquad\qquad F_H = k_H \cdot D$$

where D is the distance flown. The problem where the airline can decide $P_H, P_L, k_H, k_L$without reservation price on $P_H$, is written as

$$Max_{(P_H,P_L,k_H,k_L)}\ \pi = N_L\big(P_L - C_L(k_L \cdot D)\big) + N_H(P_H - \ C_H(k_H \cdot D))$$

$$\theta_{\ H} V(k_H \cdot D) - \alpha P_H \geq \theta_H V(k_L \cdot D\ ) - \alpha P_L$$

$$\theta_L V(k_L \cdot D\ ) - P_L \geq \theta_L V(k_H \cdot D) - P_H$$

$$U_0 + \theta_L V(k_L \cdot D\ ) - P_L \geq 0$$

$$k_H, k_L \geq 0$$

The fact that this program will, ex-ante, deliver the same rewards and fares as the optimal program can be easily seen because, if under the optimal program the airline chose $({F_H}^*, {F_L}^*, {P_L}^*, {P_H}^*)$, it only needs to set $k_H^* = \frac{{F_H}^*}{D}$, $k_L^* = \frac{{F_L}^*}{D}$. Using these equalities, we can rewrite the conditions for the optimal rewards level:

$$\frac{\theta_H}{\alpha} V'(k_H \cdot D) = \ C'_H(k_H \cdot D) \tag{15}$$

$$\theta_L V'(k_L \cdot D) = \frac{{C_L}'(k_L \cdot D)}{1 - \frac{N_H}{N_L}\frac{\left(\frac{\theta_H}{\alpha} - \theta_L\right)}{\theta_L}} \tag{16}$$

The last two expressions imply in general that $k_L < k_H$: for example, if the marginal cost of providing rewards is constant and equal for the business and leisure passengers, then

$$V'(k_L \cdot D) = \frac{{C_L}'(k_L \cdot D)}{\theta_L\left(1 - \frac{N_H}{N_L}\frac{\left(\frac{\theta_H}{\alpha} - \theta_L\right)}{\theta_L}\right)} > \frac{\alpha}{\theta_H} C'_H(k_H \cdot D) = V'(k_H \cdot D)$$

And using $V' > 0, V'' < 0$, we obtain $k_L < k_H$

Note that, if for the given parameter values, it was more profitable under the optimal program to shut down the leisure class, it will be also the case here. And in that case, the airline will choose the same fare $P_H^{**}$ while setting $k_H^{**} = F_H^{**}/D$. In other words, irrespective of whether separation or class shutdown is better, the per-distance program can mimic the optimal program ex-ante.

## 4.2 Rewards per Dollar

We now focus in the program where the rewards are based on the fare paid, that is

$$F_L = k_L \cdot P_L \qquad\qquad F_H = k_H \cdot P_H$$

The problem where the firm can decide $P_H, P_L, k_L, k_H$ without reservation price on $P_H$, is written as

$$Max_{(P_H,P_L,k_L,k_H)}\ \pi = N_L\big(P_L - C_L(k_L \cdot P_L)\big) + N_H\big(P_H - C_H(k_H \cdot P_H)\big)$$

$$\theta_H V(k_H \cdot P_H) - \alpha P_H \geq \theta_H V(k_L \cdot P_L) - \alpha P_L$$

$$\theta_L V(k_L \cdot P_L) - P_L \geq \theta_L V(k_H \cdot P_H) - P_H$$

$$U_0 + \theta_L V(k_L \cdot P_L) - P_L \geq 0$$

$$k_L, k_H \geq 0$$

Again, if under the optimal program the airline chose $(F_H{}^*, F_L{}^*, P_L{}^*, P_H{}^*)$ given again by equations (10), (11), (12) and (13), it only needs to set $k_H^* = \frac{F_H{}^*}{P_H{}^*}$ , $k_L^* = \frac{F_L{}^*}{P_L{}^*}$ , and so this program is also equivalent ex-ante to the optimal program. It is easy to show that if V is 'concave enough', then $k_L^* < k_H^*$ (see Appendix C[10]).

Note again that, if for the given parameter values it was more profitable under the optimal program to shut down the leisure class, it will be also the case here. And in that case, the airline will choose the same fare $P_H^{**}$ while setting $k_H^{**} = F_H^{**}/P_H^{**}$. In other words, irrespective of whether separation or class shutdown is better, the per-dollar program can also mimic the optimal program ex-ante.

# 5. REAL PROGRAMS EX - POST

The 'ex-post' setting considers that, after the design phase, parameter values may end-up differing from its expected values. In the optimal program, the airline would be able to adjust both rewards and fares, yet in real programs, we assume the airline is tied to the ex-ante program design, that is, the airline cannot change the percentages in the per-distance program or the fare-multipliers in the per-dollar program. Thus, in the ex-post setting the airline can imperfectly adjust values of fares and reward. To analyze this, we repeat the comparative statics exercise.

## 5.1 Rewards per Distance

Since the contract between the airline and consumers is now set, we consider that the parameter values change marginally, but the airline has already declared their multipliers for rewards, and the distances are obviously fixed. The most obvious implication is that the airline would like to change the rewards but it cannot. The airline thus solves:

$$Max_{(P_H,P_L)}\ \pi = N_L\left(P_L - C_L\big(\overline{k_L} \cdot D\big)\right) + N_H\big(P_H - C_H\big(\overline{k_H} \cdot D\big)\big)$$

$$\theta_H V\big(\overline{k_H} \cdot D\big) - \alpha P_H \geq \theta_H V\big(\overline{k_L} \cdot D\big) - \alpha P_L$$

[10] Appendix available at Google Drive - Appendix.

$$\theta_L V(\overline{k_L} \cdot D) - P_L \geq \theta_L V(\overline{k_H} \cdot D) - P_H$$
$$U_0 + \theta_L V(\overline{k_L} \cdot D) - P_L \geq 0$$

This problem is easy to solve once we consider that the airline can only change the fares, and that increasing fares always increase profit. Again, the active constraints are the H-type incentive compatibility and the L-type participation constraint. The optimal fares are then given by:

$$P_L^* = U_0 + \theta_L V(\overline{k_L} \cdot D) \tag{17}$$

$$P_H^* = \frac{\theta_H}{\alpha} V(\overline{k_H} \cdot D) - \frac{\theta_H}{\alpha} V(\overline{k_L} \cdot D) + P_L^* \tag{18}$$

The behavior of the fares ex-post when any of these parameters change is easy to analyze from equations (17) and (18), and are summarized in the following table, where the first column shows the response in the optimal program, and the second the response in the rewards-per-distance program. This allows us to show when the per-distance program can move, ex-post, the variable in the right direction (in green) and when it cannot (red), as compared to the optimal program.

**Table 2.** Responses ex-post for the optimal problem and rewards-per-distance program.

| $\frac{d}{dx}$ | $F_H$ | | $F_L$ | | $P_H$ | | $P_L$ | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $\leq 0$ | $= 0$ | $\geq 0$ | $= 0$ | $\leq 0$ | $\leq 0$ | $\geq 0$ | $= 0$ |
| $\theta_H$ | $\geq 0$ | $= 0$ | $\leq 0$ | $= 0$ | $\geq 0$ | $\geq 0$ | $\leq 0$ | $= 0$ |
| $\theta_L$ | $= 0$ | $= 0$ | $\geq 0$ | $= 0$ | Depends | $\geq 0$ | $\geq 0$ | $\geq 0$ |
| $N_H$ | $= 0$ | $= 0$ | $\leq 0$ | $= 0$ | $\geq 0$ | $\leq 0$ | $\leq 0$ | $\leq 0$ |
| $N_L$ | $= 0$ | $= 0$ | $\geq 0$ | $= 0$ | $\leq 0$ | $= 0$ | $\geq 0$ | $= 0$ |

## 5.2 Rewards per Dollar

We focus again in the Rewards per Dollar program. The logic is the same used in the previous subsection: the airline first solves this unrestricted problem, but then is locked to the problem where it cannot move the multipliers $k_L, k_H$ anymore, that is:

$$Max_{(P_L,P_H)}\ \pi = N_L\left(P_L - C_L(\overline{k_L} \cdot P_L)\right) + N_H\left(P_H - C_H(\overline{k_H} \cdot P_H)\right)$$

$$\theta_H V(\overline{k_H} \cdot P_H) - \alpha P_H \geq \theta_H V(\overline{k_L} \cdot P_L) - \alpha P_L \tag{19}$$

$$\theta_L V(\overline{k_L} \cdot P_L) - P_L \geq \theta_L V(\overline{k_H} \cdot P_H) - P_H \tag{20}$$

$$U_0 + \theta_L V(\overline{k_L} \cdot P_L) - P_L \geq 0 \tag{21}$$

The main difference here, compared to the per-distance program, is that rewards do change when fares change, albeit at a fixed rate. This problem is not as easy to solve, because we cannot easily say which restriction is active for any set of parameters $(\theta_H, \theta_L, N_H, N_L, \alpha)$: in the optimal program fares and rewards can be changed independently, leading to only two out of four constraints being active in the optimum. In the ex-post per-distance program, only fares can be changed, which again makes it obvious that those same constraints are the ones active in the ex-post optimum. But here, fare and rewards move simultaneously at a fix rate, and thus which restriction is active at the optimum is far from obvious.

What we do, then, is start from the ex-ante problem, where the H-type incentive compatibility and the L-type participation constraint are active, and analyze small changes over the set of parameters, and their impact over the restrictions. We then try to move fares and rewards in the same directions that the optimal program would while satisfying all constraints, which lead to, in some cases, conditions on functional forms and evaluation points. All calculations are mapped in Appendix D.

We show that under the following condition:

$$V'(F_H^*) < \frac{\alpha}{\theta_H k_H} \tag{22}$$

the airline can move $F_H$ (and thus, $P_H$) in the same direction of the optimal program when facing small changes over $\alpha$ or $\theta_H$ . If we have the stronger conditions

$$V'(F_H^*) < \frac{\alpha}{\theta_H k_H} - \delta\left(\frac{1}{\theta_L k_L} - \frac{\alpha}{\theta_H k_L}\right) \tag{23}$$

$$V'(F_L^*) \in \left[\frac{1}{\theta_L k_L}, \frac{1}{\delta}\left(\frac{\alpha}{\theta_H k_H} - V'(F_H^*)\right) + \frac{\alpha}{\theta_H k_L}\right] \tag{24}$$

then the airline would also be able to also move $F_L$ (and thus, $P_L$) in the same direction of the optimal program, where $\delta$ is the magnitude of the movement over $F_L$ compared to the magnitude of the movement over $F_H$. It is also easy to show that if $U_0$ is large enough, (23) and (24) are naturally satisfied given the concavity of $V$.

The same conditions would allow the airline to move $F_L$ (and thus, $P_L$) in the same direction of the optimal program when facing small changes over $\theta_L$, but this requires in the ex-post setting to move $P_H, F_H$ in the opposite direction of the movement in $F_L, P_L$, even though in the optimal program the airline would like to maintain the reward assigned to business passengers.

Finally, we show that under excluding conditions, the airline can move fares and rewards in a similar way to the optimal program when facing changes over the number of passengers of high class $N_H$ or low class $N_L$. To be more precise, if we have

$$V'(F_L^*) \geq \frac{1}{\theta_L k_L} \tag{25}$$

as in the lower bound of (24), then the airline can react correctly to changes over $N_L$, but not over $N_H$. On the other hand, if we have the opposite inequality, then the airline can react correctly to changes over $N_H$, but not over $N_L$.

A summary table is provided, where the first column show the response in the optimal program, and the second the response in the rewards-per-dollar program. Green cells indicate that the change is in the same direction as in the optimal program, red cells indicate that it is not.

**Table 3.** Responses ex-post for the optimal problem and rewards-per-dollar program.

| $\frac{d}{dx}$ | $F_H$ | | $F_L$ | | $P_H$ | | $P_L$ | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $\leq 0$ | $\leq 0$ | $\geq 0$ | $\geq 0$ | $\leq 0$ | $\leq 0$ | $\geq 0$ | $\geq 0$ |
| $\theta_H$ | $\geq 0$ | $\geq 0$ | $\leq 0$ | $\leq 0$ | $\geq 0$ | $\geq 0$ | $\leq 0$ | $\leq 0$ |
| $\theta_L$ | $= 0$ | $\leq 0$ | $\geq 0$ | $\geq 0$ | Depends | $\leq 0$ | $\geq 0$ | $\geq 0$ |
| $N_H$ | $= 0$ | $\geq 0$ | $\leq 0$ | $\leq 0^*$ (=0) | $\geq 0$ | $\geq 0$ | $\leq 0$ | $\leq 0^*$ (=0) |
| $N_L$ | $= 0$ | $\leq 0$ | $\geq 0$ | $\geq 0^*$ (=0) | $\leq 0$ | $\leq 0$ | $\geq 0$ | $\geq 0^*$ (=0) |

### 5.3 Class-shutdown comparison

We now compare class-shutdown conditions ex-post. Suppose that ex-ante, for expected values of the parameters, the airline decides to design its program to separate but, when facing ex-post parameter values, the airline has to reassess. The general shutdown condition is:

$$N_L\big(P_L^* - C_L(F_L^*)\big) + N_H\big(P_H^* - C_H(F_H^*)\big) < N_H(P_H^{**} - C_H(F_H^{**}))$$

Where * denotes separation values and ** denotes what would be chosen if shutting down the L-type class. For the case of the per-distance rewards program tough, ex-post only the fare can be adjusted, not the reward and, therefore, $F_H^* = F_H^{**}$. This implies that the shutdown condition for the per-distance program in the ex-post setting is:

$$N_L\big(P_L^* - C_L(F_L^*)\big) < N_H(P_H^{**} - P_H^*) \tag{26}$$

In the per-dollar program, however, ex-post, we have $F_H^* = k_H^* P_H^* < k_H^* P_H^{**} = F_H^{**}$. Thus, the shutdown condition for the per-dollar program in the ex-post setting is:

$$N_L\big(P_L^* - C_L(F_L^*)\big) \leq N_H(P_H^{**} - P_H^*) - \underbrace{N_H\big(C_H(k_H^* P_H^{**}) - C_H(k_H^* P_H^*)\big)}_{+} \tag{27}$$

We can easily see that this condition is stronger than the one in the rewards per-distance program in (27): charging a higher price $P_H^{**} > \frac{F_H{}^*}{k_H^*}$ to business passengers has a downside for the airline as they have to offer the correspondent rewards and face a cost $C_H(k_H^* P_H^{**}) > C_H(F_H{}^*)$. Hence, the models predict that in a per-dollar program, class shutdown will be less common than in a per-distance program. This matches the data we collected; we followed four pairs of local city pairs in Chile, Canada and the U.S. on October and November of 2014 and discovered that LATAM and Air Canada, both with per-distance programs often closed the lower classes. LATAM closed the fourth, third and second tier class 68%, 53% and 26% of the time respectively, while Air Canada closed the third and second tier class 34% and 4% of the time respectively. On the other hand, Southwest, with a per-dollar program, closed its lowest class only once, while the middle class was never shutdown.

## 6. CONCLUSIONS

Since their inception in the 80s, airlines' frequent flyer programs (FFP) have become one of the largest customer loyalty programs in the world economy. However, despite their size and importance, these programs have remained relatively unexplored from a microeconomics point of view. Basso et al (2009) study the FPP considering the existence of a business related traveler, where the employer is who actually pays the ticket, which impose a moral-hazard situation where the employee might not always choose the cheapest ticket but will try to maximize individual benefits and it is there where FFP rewards play a major role.

We extend this approach, proposing a theoretical model that considers the existence of two types of travelers, business and leisure, who are not distinguishable by the (monopoly) airline ex-ante, and which is the source of an adverse selection problem, and where business travelers do not pay fully their air tickets, inducing a moral hazard problem between them and their employers, who actually pays. The airline then faces a second-degree price discrimination problem, where it attempts to separate traveler types with the use of different menus of prices and rewards, while exploiting the moral hazard problem by 'bribing' business travelers.

We find that the moral hazard problem induces an upward distortion in rewards for business travelers (distortion at the top) while deepening the typical distortion at the bottom caused by adverse selection (even smaller rewards for leisure travelers).

We compare this 'optimal' program, where fares and rewards can be chosen independently, to real-life programs, where rewards are linked either to distance travelled or to the fare paid. We show that, at the design stage of the program, when demand parameter values are taken at its expected values (the ex-ante setting), the three programs are equivalent. Yet, ex-post, once the programs have already been designed and they cannot be modified, if demand parameter values differ from expected values, nor the per-distance nor the per-dollar program can mimic the optimal program. We show, however, that the per-dollar program allows the (monopoly) airline to respond to changes in a closer way to the optimal program that the per-distance program, which is consistent with the recent changes to per-dollar programs of a number of legacy carriers. Finally, we show that, ex-post, it is more likely that the lower classes are shut-down in a per-distance than in a per-dollar program, which is consistent with the data we collected from LATAM Airlines, Air Canada (rewards per distance) and Southwest Airlines (rewards per dollar).

The model introduced in this study could be easily used for future research. We focused in a single monopoly airline serving the demand, so a natural step would be to include competition:
Basso et al (2009) show that for the case of only business travelers, a lone airline can indeed take advantage of the moral hazard, increasing prices up to the employers' reservation price and defeating the competition. Yet, if all airlines use FFPs, rewards become simply a new competitive instrument which enables tougher competition: airlines will dissipate profits through FFP rewards in a standard Prisoner's Dilemma fashion. Prices will remain high though, showing that all benefits

flow from employers and airlines to employees. These conclusions could translate to the two-classes case, generating different conditions under which an airline could find beneficial to use one or the other scheme of rewards.

Also, in this study we showed the results for only one homogenous market. Yet, when designing their programs, airline may need to use single FFP contracts (i.e. just one pair of multipliers) for many different city pair markets, with different distances and number of passengers of each class. It could be the case then, that one real program may be preferable for some circumstances of demands and markets, while the other program may be preferable for different circumstances, something worthy of research.

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