AN OLIGOPOLY MODEL FOR THE AIRLINE MARKET AND ITS RELATION TO THE ANALYSIS OF AIRPORT PRICING

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ABSTRACT

Airport pricing has begen analyzed in the literature using two approaches. The first uses a classical partial equilibrium model where the demand for airports depends on congestion costs of both passengers and airlines. The second models rather simple oligopolistic airline markets, and calculates the additional toll that airlines should be charged to attain maximization of social welfare. The two approaches have not only examined fairly different questions –the first has significantly focused on capacity and ownership issues as well–, but also have grown somewhat disconnected. By building a larger model where both approaches can be fitted, it is shown that abstracting from the airline market, as it is done in the first approach, is not a reasonable approximation because it overlooks a crucial fact, namely that airport services are an input for the production of an output: travel. By formally modeling equilibrium –conditional on airport's decisions– in an oligopolistic airline market, it is shown that what would have been called social welfare in the first approach in fact never correspond to the sum of airline profits and consumers' surplus. This has important consequences for the analysis of airport markets.

1. INTRODUCTION

The airport pricing analyses put forward in the last 20 years can be roughly divided into two broad approaches: the more popular *one market approach* and the newer *airline market approach.* The *one market approach* follows a classical partial equilibrium analysis. It assumes that the demand for the airport is a function of a full price, which includes the airport charge and, in an additive fashion, some cost measure of the delay caused by congestion; delay is assumed to affect both airlines and passengers. More importantly, *consumer surplus* is measured by integration of the airport's demand. Examples of this approach are Morrison and Winston (1989), Zhang and Zhang (1997), Carlsson (2003) and Oum et al. (2004). It has been used to analyze many different issues such as congestion pricing, optimal capacity, effects of privatization and efficiency of regulation schemes. The *airline market approach* is newer and, therefore, has a smaller number of papers (e.g. Brueckner, 2002; Pels and Verhoef, 2004; Raffarin, 2004). In this approach the airline market is formally modeled as an oligopoly –albeit in fairly simple ways–, which takes airport charges as given. Airports however, are not always considered integrally; in some cases only public airport authorities, who have to set a tax to be paid in addition to the airport charge –implicitly assumed to be marginal cost–, are considered. In these cases, airport profits do not enter the social welfare function. This approach has been driven by increasing levels of delays at hubs throughout the world and hence its focus has been narrower: it has looked at optimal runaway pricing under congestion but not at other issues such as privatization or optimal capacity.

It is clear that the two approaches are rather different and, in fact, some authors using the second approach have been somewhat critic of the first one. For example, Raffarin (2004) says that it is rather strange that the pricing rules obtained from the *one market approach* do not consider passengers' utility. This is not completely accurate though. Passengers are indeed somehow considered in the approach as delay costs affect them as well, something that Raffarin missed. But what it is true is that a more rigorous vision of the problem must recognize that airports provide an input –airport services–, which are necessary for the production of an output –displacement– that is sold at another market. When looking at the problem in this vertical input-output framework, three and not only two actors are neatly distinguished: airlines, passengers and airports. Some questions arise naturally. Who are the *consumers* of airport services? How should we measure their surplus? How does airport's demand depend on the structure of the downstream market? In short, an important question is how the *one market approach* and the *airline market approach* relate to each other. This is what is analyzed here and it is important because the two approaches have studied rather different issues. The analysis is done by, first, proposing an oligopoly model for the airline market and studying its main properties (section 2) and, second, using it to unveil the differences between the two approaches (section 3). It is shown that a rigorous analysis of airport pricing simply cannot avoid formal modeling of the downstream market. In particular, what is called maximization of social welfare in the *one market approach* in fact does not correspond to it, something important to reveal since the public airport case lies at the core, as a benchmark, in all studies. Moreover, the remarks and techniques used here are not limited to airports but can be applied to any transportation port or terminal, since the situation is essentially the same.

2. THE AIRLINE MARKET

The game I look at is a two stage game: first, airports choose a price per take-off/landing, P, and a capacity, K, then airlines –in a oligopolistic fashion– choose their quantities. In this paper, I am only interested in Nash Equilibria of the airlines' sub-game. The following simplifying assumptions are made: (i) abstraction from network and route structure decisions by having only two national airports, that is, two origin-destination pairs (ii) there is only demand for round trips, not unidirectional trips, which implies that airlines are monoproducers.[1] There are N airlines with identical cost functions, facing differentiated demands in a non-address setting with fixed variety: differentiation is thus horizontal and N is exogenous, representing the main airline industry structure indicator in the model. Each firm's demand depends on a vector of full prices, $\boldsymbol{\theta}$:

$$q_i(\boldsymbol{\theta}) = q_i(\theta_i, \boldsymbol{\theta}_{-i})$$
$$\theta_i = t_i + G(\tau_i) + \alpha\big(D(Q,K_1) + D(Q,K_2)\big) \qquad (1)$$

where q_i is the demand faced by airline i, Q_i is the number of flights of airline i, Q is the total number of flights of all airlines, t is the ticket price for the round trip, $G(\tau)$ is schedule delay cost, τ is the expected gap between passengers' actual and desired departure time, $D(Q,K_h)$ represents flight delay because of congestion at airport h and α represents passengers value of time. Note that τ depends on the frequency chosen by the airline; the higher the frequency, the smaller the gap. Thus, schedule delay cost can be written as $g(Q) \equiv G(\tau(Q))$ where $g'(Q)<0$ while $g''(Q)$ has no evident sign a priori.[2] The delay function is going to be given by[3]

$$D^h(Q,K_h) = Q/\big(K_h(K_h - Q)\big) \qquad (2)$$

In a round trip, passengers have one take-off and one landing in each airport, which is why there are two capacities considered; D^h in (2) represents total delay of both take-off and landing at each airport h, which requires to assume that capacities of take-off and landing are equal. Assuming that demands are linear, symmetric and that outputs are substitutes:

$$q_i(\boldsymbol{\theta}) = a - b\theta_i + \sum_{j\neq i}^{N} e\theta_j \qquad (3)$$

where a, b and e are positive. Inverting the system and re-labeling the parameters I get

$$\theta_i = A - B\cdot q_i - \sum_{j\neq i}^{N} E\cdot q_j \qquad (4)$$

where A, B and E are positive. As in Vives (1985), A, B and E are assumed to be fixed and $B>E>0$, that is, outputs are imperfect substitutes. It is easy to verify that $B>E$ is equivalent to $b>(N-1)e$, that is, if all full prices increase by the same amount, the demand for airline i will decrease. This condition is sometimes called *diagonal dominance*.

[1] These assumptions are consistent with Pels and Verhoef (2004) and are, in fact, a generalization of Brueckner (2002) and Raffarin (2004) who consider a single airport.

[2] Schedule delay cost represents the monetary value of the time between the passenger's desired departure time and the actual departure time. It was first introduced by Douglas and Miller (1974).

[3] This function was proposed by the US Federal Aviation Administration (1969) and is discussed in Horonjeff and McKelvey (1983). It has been used by Morrison (1987), Zhang and Zhang (1997), Carlsson (2003) and Oum et al. (2004).

We will assume that airlines behave as Cournot oligopolists, i.e. they choose quantities. This assumption is backed by some empirical evidence (e.g. Brander and Zhang, 1990) and has been the standard assumption in airline oligopoly models (e.g. Brueckner and Spiller, 1991). The differentiated linear demands model was chosen because, without congestion and schedule delay cost –i.e. when $t_i \equiv \theta_i$ – and with constant marginal cost, the game is well behaved in that a unique equilibrium exists and, as N increases, Cournot-Nash equilibrium prices approach marginal cost, i.e. market power decreases with the number of firms (Vives, 1985). So, in principle, the model will be useful to assess the importance of the airline industry structure (N) in airport pricing. We discuss here how these properties change when there is congestion and schedule delay cost.[4]

A few more comments about the demand model are worthy to be spelled out. First, a simpler assumption may have been homogeneity in the Cournot competition, as it has usually been done (e.g. Pels and Verhoef, 2004). In that case, total inverse demand is given by $\theta(q) = A - \sum_{i=1}^{N} B \cdot q_i$ and, because in homogenous Cournot prices are equal, each firm's inverse demand is $\theta_i = A - B \cdot q_i - \sum_{j \neq i}^{N} B \cdot q_j$. This last expression is obviously equal to (4) when B=E. Assuming homogeneity though is unnecessary and hence it is avoided. All results will be expressed in terms of A, B and E so, to recover the homogenous case it will suffice to replace E by B: homogeneity will be a particular case of this model.[5] Second, in this demand model we have incorporated schedule delay cost, something that has not been done in the *airline market approach* to airport pricing. Schedule delay is an important aspect of service quality and hence we consider it[6]. Finally, the fact that the number of active firms in the airline market is fixed and not defined within the model may appear to be inconsistent with longer run decisions; it is usually assumed that a fixed N can only provide predictions about short run behavior. N is a parameter here because it may not be realistic to assume that it is defined only through free entry. In fact, airports may have preferences regarding N that are different than the free entry equilibrium and they may indeed have a sizeable influence on the number of active firms. Anyhow, the equations that define the free entry N are identified, and I show that this N is finite.

Using (1) and (4), the following system of *inverse demands* faced by the airports is obtained: $t^i = A - B \cdot q_i - \sum_{j \neq i}^{N} E \cdot q_j - g(Q_i) - \alpha\left(D(Q, K_1) + D(Q, K_2)\right)$. Things can be simplified though, by recognizing that q_i and Q_i are related through aircraft size and load factor, that is, $q_i = Q_i \times$ Aircraft Size $\times$ Load Factor. Here, we impose that the product between aircraft size and load factor, denoted by S, is constant. This effectively makes the vertical relation between airports and airlines of the fixed proportions type.[7] A variable proportions case would arise if, before a change in airport charges, airlines decide to change S (aircraft size, load factor or both). Thus

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[4] The fact that prices decrease with N is true not only for linear demands but for any other system of demands, as long as: (i) inverse demands have slopes that are bounded as N increases (ii) total demand is bounded when N goes to infinity, even when all prices are zero; in other words, demand for each product is smaller when the number of product increases. Since travel is a derived demand, assumption (ii) is reasonable here.

[5] The system of inverse demands in the homogenous case is not invertible so that a counterpart to (3) does not exist: some calculations have to be done differently that in the differentiated case. It is still true though that, in the results, it is enough to replace E by B to recover the homogenous case.

[6] It was considered in models of airline competition: analytically by Oum et al. (1995) and empirically measured by Morrison and Winston (1989).

[7] This assumption was also made by Brueckner (2002) and Pels and Verhoef (2004).

$$t^i(Q_i,\mathbf{Q}_{-i}) = A - SBQ_i - \sum_{j\neq i}^{N} SEQ_j - g(Q_i) - \alpha\big(D(Q,K_1)+D(Q,K_2)\big) \tag{5}$$

It can be noted that linear demands in full prices do not lead to *inverse demands* that are linear in output, as D is not linear and there is no reason to think that g is.[8] Also, equation (5) show a potential source of problems: one of the main points of using this model is to analyze the influence of airline market structure (N). As explained, when there is no congestion and schedule delay cost, market power decreases with N but here, intuitively, as N grows to infinity, the number of flights produced by each carrier goes to zero –while the total number of flights would not– making schedule delay costs explode. This happen because this cost depends only on the schedule delay each firm induces, so $g(Q_i)\rightarrow\infty$ when $Q_i\rightarrow 0$. The inverse demand function will then not be defined (positive) for values of Q_i, given $\mathbf{Q}_{-i}$, close to zero and will probably be upward-sloping before becoming downward-sloping for higher values of Q_i. To see this analytically, I make now assumptions regarding schedule delay cost that will be useful later. I assume that (a) The monetary cost of the gap between actual and desired departure time, τ, is proportional to its length, and (b) τ is inversely proportional to the frequency of flights. Assumption (a) is similar to what has been already assumed regarding congestion delay costs. As for assumption (b), note that the interval between flights is given exactly by the inverse of the frequency; therefore assumption (b) is equivalent to say that τ –the expected gap between passengers' desired and actual departure– is directly proportional to the interval between flights (in the fixed proportions case –S constant–, frequency is directly given by Q_i) Hence, if assumptions (a) and (b) are maintained, we obtain that $g(Q) = G(\tau(Q)) = \gamma\cdot\tau(Q) = \gamma\cdot\eta\cdot Q_i^{-1}$, where γ is the constant monetary value of a minute of schedule delay and η is a constant.[9] The potential problem described does indeed arise: the residual inverse demand, $t^i(Q_i,\mathbf{Q}_{-i})$, is first negative and upward-sloping, it then becomes positive and finally downward sloping, when the linear part of the function starts dominating schedule delay cost. For higher values of Q_i, congestion starts to kick in which makes t^i decrease faster than linearly (see figure 1). Thus, schedule delay cost put by itself, and regardless of other technological considerations, a limit to the number of active firms in the industry, implying that the perfect competition case is not consistent with this model.

The final ingredient necessary, before analyzing equilibria, is costs. Airline costs are

$$C_A^i(Q_i,\mathbf{Q}_{-i},P_h,K_h) = Q_i\left[c + \sum_{h=1,2}(P_h + \beta D(Q,K_h))\right] \tag{6}$$

[8] Pels and Verhoef (2004) obtain linear *inverse demands* because they assumed a linear delay function and no schedule delay cost.

[9] If passengers' desired departure time is uniformly distributed along the day, then assumption (b) holds and η=1/4. Note that we are assuming only one period here and not peak and off-peak periods: this is a model of congestion pricing and not peak-load pricing. If we were to assume more than one period (e.g. Zhang and Zhang, 1997), it would still be a reasonable assumption that, within each period, desired departure times are uniformly distributed.

The term in square brackets on the right hand side represents cost per flight (round trip) which includes pure operating costs c, airports charges P_h and congestion delay costs.[10] Using the expression for delay in (2), it is easy to verify that marginal costs are strictly increasing and larger than average cost. Cost, marginal cost and average cost functions are strictly convex. Airline profits are then obtained as the difference between revenues, $t^i q_i = t^i Q_i S$, and costs. Using (5) and (6) and reordering we obtain that airline i's profits are given by

$$\phi^i(Q_i, \mathbf{Q}_{-i}, P_h, K_h) = \left[AS - BQ_i S^2 - \sum_{j\neq i} EQ_j S^2 - c - \sum_{h=1,2} P_h \right] Q_i - SQ_i g(Q_i) \\ - (\alpha S + \beta) \sum_{h=1,2} Q_i D(Q, K_h) \tag{7}$$

Regarding existence, unicity and stability of Cournot-Nash equilibria, it can be shown that under assumptions (a) and (b) about schedule delay cost, $\phi^i_{ii} < 0$ so existence is guaranteed (as long as the solution is interior which we assume for now). Uniqueness and symmetry is ensured because the best reply correspondences, $\Psi_i(\mathbf{Q}_{-i})$, defined by $\phi^i_i(\Psi_i(\mathbf{Q}_{-i}), \mathbf{Q}_{-i}) = 0$, are actually continuously differentiable functions of the sum of quantities of other firms, that is, $\Psi_i(\mathbf{Q}_{-i}) = \Psi_i\left(\sum_{j\neq i} Q_j\right)$, and their slopes with respect to $\sum_{j\neq i} Q_j$ are greater than -1 (see theorem 2.8 in Vives, 1999). So, despite the rather unusual inverse demands, the problem is well behaved in the sense of existence and unicity. As for Cournot (or *tatonnement*) stability, it has been proved that a sufficient condition for stability is that the best reply mapping is a contraction, condition that holds here only for N very small (2 or 3). This is a well known problem in the case of homogenous Cournot and while differentiation ($E<B$) gives us some latitude, congestion works in the opposite direction. Stability could be achieved by imposing more structure on the parameters, but that would require substitutability (E) to decrease with N.

$\phi^i_i = 0$ $\forall i$, gives us the unique and symmetric Cournot-Nash equilibrium of the game. Imposing symmetry, we obtain the following important equation

$$\Omega(Q, P_h, K_h, N) = (\alpha S + \beta) \sum_{h=1,2} \left(D^h(Q, K_h) + (Q/N) D^h_Q(Q, K_h) \right) + S\left(g(Q/N) + \frac{Q}{N} g'(Q/N) \right) \\ + (Q/N) S^2 \left(2B + (N-1)E \right) + c + \sum_{h=1,2} P_h - AS = 0 \tag{8}$$

Equation (8) implicitly define a function $Q(P_h, K_h; N)$, that is, airports' demand as a function of airport charges, capacities and airline market structure, N.[11] Three observations can be made:

[10] Note that here it is assumed that c does not depend on aircraft size or load factor, which may appear as a strong simplification. Given that Q_i is proportional to q_i, this essentially says that the cost per passenger is fixed, something that has been assumed elsewhere (e.g. Brander and Zhang, 1990, Pels and Verhoef, 2004). Alternatively, one could assume that a single aircraft size and load factor prevail, as in Brueckner (2002). Also, the cost function should depend on a vector w of other input prices. That dependence could be modeled through $c(w)$. Since it is assumed that input prices other than airport charges are constant throughout, vector w will be suppressed for notational simplicity.

[11] For $0<Q<K_h$, it is easy to check that Ω has continuous partial derivatives with respect to Q, K_h, P_h and N and that Ω_Q is non-zero. Thus, the implicit function theorem holds.

first, if assumptions (a) and (b) hold, then $g(x)+xg'(x)=0$ so the second term would be zero. Second, one can define without loss of generality $P=P_1+P_2$; if airports were to be priced jointly then an explicit expression of the airports' inverse demand $P(Q,K_h;N)$ is obtainable. Third, the subgame equilibrium $Q^i(P_h,K_h;N)=Q(P_h,K_h;N)/N$ induces subgame equilibrium air fare and delay, simply by replacing equilibrium values in t^i and D (equations 5 and 2).

Before moving into comparative statics analyses, let me look at the free entry long run equilibrium. It is obtained when the revenue per flight, $S\cdot t^i(Q_i,\mathbf{Q}_{-i})$, equals average cost (profits are zero). Formally, with free entry

$$AS-\big(B+(N-1)E\big)S^2\frac{Q}{N}-Sg(\frac{Q}{N})-c-\sum_h P_h-(\alpha S+\beta)\sum_h D(Q,K_h)=0 \qquad (9)$$

Equations (8) and (9) together determine the free entry equilibrium $Q(P_h,K_h)$ and $\overline{N}(P_h,K_h)$. To see this equilibrium graphically, first note that under (a) and (b), the marginal revenue of each firm, $MR_i(Q_i,\mathbf{Q}_{-i})$, is decreasing in Q_i (recall that $\phi^i_{ii}<0$ and that airline's cost are convex). Further, it intersects the inverse demand function for flights, $S\cdot t^i(Q_i,\mathbf{Q}_{-i})$, which is first increasing and then decreasing, at its maximum. Next, both marginal and average cost functions are convex and increasing, the former being larger than the latter. Therefore, the free entry equilibrium is as in figure 1. $\overline{N}$ is given by $Q/\tilde{Q}_i$, where $\tilde{Q}_i$ is determined by the profit maximization first order condition *marginal revenue equals marginal cost* and the zero profit condition *average cost equals revenue per flight*. At this point, average cost and $S\cdot t^i$ are tangent. Thus, schedule delay cost plays a role similar to a U-shaped average cost function when demand is downward sloping, in putting a limit to the number of firms that can be active in the Cournot oligopoly. As explained, in this paper we are interested in N as a parameter for the airline market, so we assume that $N<\overline{N}$. When this is true, $Q_i>\tilde{Q}_i$ (we will see shortly that equilibrium Q_i is decreasing with N) and therefore a sufficient condition for an interior solution is $N\le\overline{N}$, which should always hold.

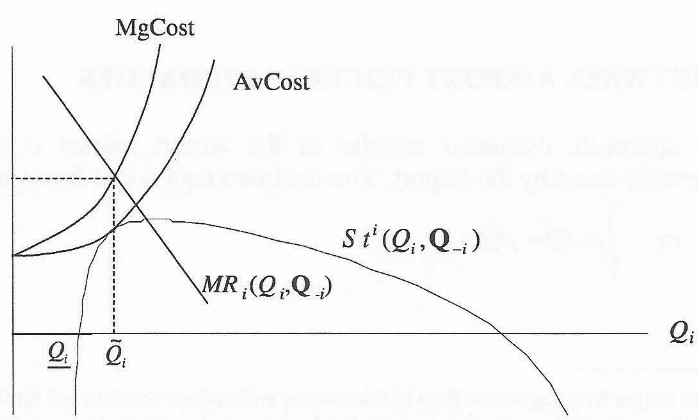

Figure 1: Free entry long run equilibrium

We can now turn to comparative statics. Here we are interested in learning how airports' demand changes with P_h, K_h and N or, alternatively, how the inverse demand $P(Q, K_h; N)$ changes with Q, K_h and N. Let us first look at how Q changes with N. If assumptions (a) and (b) hold, we have

$$\frac{dQ}{dN} = -\frac{\Omega_N}{\Omega_Q} = \frac{Q}{N}\left(\frac{(\alpha S + \beta)\sum D_Q^h + S^2(2B - E)}{(\alpha S + \beta)\sum D_Q^h + S^2(2B - E) + (\alpha S + \beta)\sum (ND_Q^h + QD_{QQ}^h) + S^2 EN}\right) \quad (10)$$

It is direct to check that dQ/dN is positive. Also $d^2Q/dN^2 < 0$, $\forall E \in (0,B]$. So, total flights increase with N at a decreasing rate. But even if we were to allow N to increase without bound, by assuming schedule delay cost away, the total number of flights would not diverge (the limit of Ω when $N \to \infty$ and $g=0$ lead to a well defined implicit function Q). Also, because $Q/N > dQ/dN$, we have that $dQ^i/dN < 0$ so that each firm's number of flights decrease with N. In the absence of congestion, when $E \to 0$, Q^i becomes independent of N: each firm has its own turf.[12] With congestion, Q^i decreases even when substitutability is very low ($E \to 0$) because the congestion externality causes marginal costs to increase. Note that this analysis, and any other examination of changes with respect to N, are valid in the subgame only because P and K are fixed and not functions of N. In the overall game they will be. All other derivatives, such as $\partial Q / \partial P_h$, $\partial Q / \partial K_h$ and $\partial^2 Q / \partial P_h^2$ are obtained in a similar fashion as above. They are not derived here because they are not central for what follows. They are essential for the analysis of airport markets of course, but this is done elsewhere (Basso, 2005a).

Finally, regarding how market power (air tickets) change with N, Vives (1985) result that $dt^i/dN < 0$ when $E \in (0,B]$ is the normal case here, but it can be shown that the opposite case may also arises. This occurs because when substitutability is low (E<<B), the overall increase in Q with N does not bring prices down; Q^i may still strongly decrease though, because marginal cost went up due to increased congestion, raising the price. When E is close to zero and if B is large, this effect may dominate the demand effect –inward shifting due to increased congestion– and, as a result, prices may actually increase with N.[13] When there are no externalities and no substitutability (E=0), a change in N does not affect marginal cost or demand –each firm is effectively acting as a monopolist in its own market– which is why N does not affect prices.

3. RELATION BETWEEN AIRPORT PRICING APPROACHES

In the one market approach, *consumer surplus* in the airport market is calculated through integration of the demand faced by the airport. The next two equivalent forms have been used:

$$\int_{\rho}^{\infty} Q(\rho)\, d\rho \quad \text{or} \quad \int_{0}^{Q} \rho\, dQ - \rho Q \quad (11)$$

[12] When E=0, it is no longer true that a new firm in the market will reduce the demand for the existing ones: since there is no substitutability, every firm creates a completely new and independent market. Asymptotic results regarding prices do not hold in this case. See footnote 4.

[13] This theoretical insight is indeed confirmed by numerical simulations.

where ρ is a full price at the airport market level. As shown in the previous section, for given P_h, K_h and N, the airline market reaches an equilibrium $Q(P_h, K_h; N)$ characterized by partial derivatives such as the one in (10). The full price approach was useful to derive this equilibrium but now time costs are built inside the demand faced by the airport, Q. Therefore, equivalent expressions for (11), but within the setting of this paper, are

$$\int_P^\infty Q(P, K_h, N)\, dP \quad \text{or} \quad \int_0^Q P(Q, K_h, N)\, dQ - P(Q, K_h, N)Q \tag{12}$$

We can, without loss of generality, restrict our attention to P (recall that we defined $P=P_1+P_2$), because any later division of P into its components does not change the value of the expressions. As it is obvious, the one market approach abstracts from what happens in the airline market. The reason for this could be tracked down to informational aspects: regulators of private airports and managers of public airports would like to set regulation rules and efficient prices respectively based on information on airports' demand and cost only, avoiding all the complexities of the previous section. The question is whether this is a sensible thing to do or too much is lost in the way. What interest us in this section then, is to understand how expressions in (12) are related to the airline profits and passengers' surplus. In order to do this, we first need to calculate the variation of Marshallian passenger surplus, CS. In this case, CS is given by a line integral but it has a solution that is path independent because $\partial q_i / \partial \theta_j = \partial q_j / \partial \theta_i$. Therefore, CS is equal to the Hicksian measures (compensating and equivalent variations). Taking a linear integration path:

$$CS(P_h, K_h, N) = \int_{\theta(P,K,N)}^{A} \sum_i^N q_i(\theta) d\theta_i = \frac{(B + (N-1)E)S^2 Q(P_h, K_h, N)^2}{2N} \tag{13}$$

As shown, it does not really make sense to allow N to grow without bound unless we assume schedule delay costs away. If we do so, we get that $\lim_{N\to\infty} CS = (1/2)ES^2 Q(P_h, K_h, \infty)^2$ (recall that Q does not diverge in this case). More interesting is to see what happens with dCS/dN. It can be checked that, when there is no congestion, $dCS/dN > 0 \ \forall \ E \in (0, B]$. But, as in the case of t^i, it can be shown that when there is congestion and substitutability is small, the opposite case may occur. However, the somewhat stranger case $dCS/dN < 0$ is not necessarily tied to increasing prices as one may expect. Even less intuitive cases may arise, such as prices going down with N but consumer surplus also going down, or prices increasing with N but consumers surplus also increasing. The reason why these may happen is that, as N increase, not only quantities and prices changes but demands shift as well due to increased congestion. Thus, although prices may be smaller (larger), the area under the demand curves may have decreased (increased) as well. Simulation show that when E is close to zero, the three combinations may arise for different parameter values. When substitutability is not small, that is, when B is not much larger than E, we are back to the *normal* case $dCS/dN > 0$ and $dt^i/dN < 0$.

Now that we have calculated CS, we can disentangle what $\int_P^\infty Q(P, K_h, N)\, dP$ represents. In the sub-game equilibrium, the profit of each airline is obtained by replacing equilibrium $(Q_i, \mathbf{Q}_{-i})$ in equation (7). Since the equilibrium is symmetric, all equilibrium profits will be identical and functions of $Q(P,K_h,N)$, that is $\phi^i(P,K_h,N)=\phi^1(Q(P,K_h,N),P,K_h)\ \forall i$. Sub-game equilibrium total profits for the airline industry as a whole, Φ, are then easily calculated as $\Phi(P,K_h,N)=N\cdot\phi^1(P,K_h,N)$, that is

$$\Phi(P,K_h,N)=QS\left[A-\frac{QS}{N}(B+(N-1)E)-g\left(\frac{Q}{N}\right)-\alpha\sum D(Q,K_h)\right] \\ -Q\left[c+P+\beta\sum D(Q,K_h)\right] \tag{14}$$

Consider the total derivative of Φ with respect to P. Differentiating and using equation (8) we get

$$\frac{d\Phi}{dP}=-Q(P,K_h,N)-\frac{(N-1)ES^2Q}{N}\frac{\partial Q}{\partial P}-\frac{(N-1)}{N}(\alpha S+\beta)Q\sum_h D_Q^h\frac{\partial Q}{\partial P} \tag{15}$$

Reordering and integrating from P to ∞, we finally obtain

$$\int_P^\infty Q(P,K_h,N)\,dP=\Phi+\frac{(N-1)ES^2Q^2}{2N}-\frac{(N-1)}{N}(\alpha S+\beta)\int_P^\infty Q\frac{\partial Q}{\partial P}\left(\sum_h D_Q^h\right)dP \tag{16}$$

To better understand this result, let me first assume congestion away, which makes the third term on the right hand side vanish, and schedule delay cost away, which allow N to increase without bound. Now, $\int_P^\infty Q(P,K_h,N)\,dP=\Phi+((N-1)/2N)ES^2Q^2$ while the expression for CS in (13) remains unchanged. When N goes to infinity, i.e. the competitive case, we get $\int_P^\infty Q(P,K_h,N)\,dP=\Phi_{N=\infty}+(1/2)ES^2Q(P,K_h,\infty)^2\equiv\Phi_{N=\infty}+CS_{N=\infty}$. If N is 1, on the other hand, we obtain $\int_P^\infty Q(P,K_h,N)\,dP=\Phi_{N=1}$. That is, when $N\to\infty$ and congestion and schedule delay costs are absent, integration of the airports' demand captures both airlines' profits and consumers' surplus. When the airline market is monopolized however, integration of the airports' demand captures only the monopoly airline profits. These results were obtained by Jacobsen (1976), but here we can generalize them to the oligopoly case $1<N<\infty$. We obtain

$$\int_P^\infty Q(P,K_h,N)\,dP=\Phi+CS-\frac{BS^2Q^2}{2N} \tag{17}$$

This shows that when the airline market is an oligopoly, integration of the airports' demand completely captures airlines profits but only partially consumer surplus. If we impose homogeneity (B=E), we obtain $CS=(1/2)BS^2Q^2$ and therefore $(1/2N)BS^2Q^2=(1/N)CS$. Thus, (17) becomes $\int_P^\infty Q(P,K_h,N)\,dP=\Phi+((N-1)/N)CS$, result that is in line with Quirmbach (1984). Equation (17) though generalizes Quirmbach's result to the case of differentiated demands. More intuition is obtained by linking this to a result by Bergstrom and Varian (1985). They proved that a symmetric homogenous Cournot equilibrium would also arise if a social planner maximizes a pseudo-social welfare function given by $\pi^T+((n-1)/n)CS$, where π^T is industry-wide profits and n is the number of firms. Thus, under homogeneity, symmetry and absence of congestion and schedule delay cost, integration of the airports' demand would capture exactly the pseudo-social welfare that is maximized in the downstream market. And since in the downstream market, the more competition there is, the higher the fraction of *CS* that is considered in the pseudo-social welfare function, a conclusion is that only under high levels of competition will the integrals in (12) adequately capture the surpluses of actors in the downstream market: the less competition, the larger the divergence.[14] Two other problems arise though. First, when schedule delay cost is present, N cannot increase without bound, so competition is necessarily limited (under Cournot setting). Also, we assumed congestion away. When congestion is considered, the expression of the integral of Q is not (17) but actually

$$\int_P^\infty Q(P,K_h,N)\,dP = \Phi + CS - \frac{BS^2Q^2}{2N} - \frac{(N-1)}{N}(\alpha S+\beta)\int_P^\infty Q\frac{\partial Q}{\partial P}\left(\sum_h D_Q^h\right)dP \qquad (18)$$

Something it is clear at once: when N=1, the integral still captures the monopoly airline profits, but if $N\to\infty$ (assuming g=0), we no longer have airlines' profits plus consumers' surplus; another term appears. This shows that $\int_P^\infty Q(P,K_h,N)\,dP$ can never be considered to represent *consumer surplus* as it is done in the *one market approach.* It never adequately captures the surplus of airports' consumers (airlines and passengers): when N is small, it simply leaves consumers out of the picture, and when N is large it does incorporate them but there is another term interfering. We now know that the third term in (18) has to do with lost consumer surplus in the downstream market caused by market power. What about the fourth term? Clearly, this term has to do with uninternalized externalities in production and consumption, in this case, congestion. Its influence in airport pricing analysis is significant, as shown in Basso (2005a).

4. CONCLUSIONS

It is now clear that a correct analysis of airport pricing simply cannot avoid formal modeling of the downstream market. It is required to adequately set up the maximization of social welfare problem. This can be done correctly by either considering directly the three actors involved, or by adding the missing terms to the integral of airport's demand. The *airline market approach,* then, stands as the correct way to study airport pricing. This is bad news for managers of public airports or airport regulation authorities: even in a setting of complete information, optimal pricing and capacity will require detailed knowledge on the market structure and demand of the airline market; information on costs and demand for airports alone are not enough. This unquestionably complicates the problem to a great extent. Furthermore, as mentioned before, the *airline market approach* has examined rather different issues than the ones that have been examined with the *one market approach*: airports profits have not been always considered and capacity has always been assumed to be fixed. In this sense, the airline oligopoly model proposed in section 2 is a step forward: it may be used as the building block of a new and broader analysis of airport pricing, capacity and ownership, as it generalizes the models that are available in the literature. In fact, these generalizations –consideration of two airports, imperfect substitutability and schedule delay costs– do play important roles in understanding the main issues at the airport market level (Basso, 2005a).

[14] This linkage between input market surplus and *what is maximized downstream* is stronger though. It does generalize to many other cases through the concept of potential function of a game (Basso, 2005b).

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