THE HYPER-NETWORK EQUILIBRIUM APPROACH FOR THE LAND USE AND TRANSPORT INTEGRATED MODEL: THE CASE WITHOUT EXTERNALITIES

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ABSTRACT

The household's decisions, from their residential location to their members' trip choices, are analyzed in a long term equilibrium approach by building a model that represents all consumers' choices (location, travel and route choices) by logit or entropy models. Consumers optimize their combined residence and transport options simultaneously, which are represented as a set of paths in a hyper-network that extends the transport sub-network to represent land use and transport demand options. We study the land use and transport static equilibrium differentiating the cases without interactions among agents, presented in this paper, and the case with externalities[1]. The case without externalities assumes that the interactions in residential location are lagged in time, so such effects are exogenous for the equilibrium. In such a case the method defines a strictly convex, coercive and non-restricted objective function. The approach provides a comprehensive characterization of the solution regarding existence and uniqueness, together with an algorithm to obtain the solution with well-defined convergence properties. The model is applicable to real size problems.

Keywords: hyper networks, integrated land- use transport equilibrium, logit models.

RESUMEN

Las decisiones de los hogares, desde su localización residencial hasta los viajes que realizan sus miembros, son analizadas como un modelo de equilibrio de largo plazo que representa todas las decisiones (localización, viajes y rutas) por modelos logit o entrópicos. Los consumidores optimizan conjuntamente su localización residencial y opciones de transporte, representadas por un conjunto de rutas en una hiper-red donde se extiende la sub-red de transporte para representar el uso de suelo y las opciones de transporte disponibles. Se estudia el equilibrio estático con congestión diferenciando los casos sin considerar interacción entre agentes, presentado en este trabajo, y el caso con externalidades[2]. El caso sin externalidades asume que esas interacciones están desplazadas en el tiempo, luego tales efectos son exógenos al equilibrio. En este caso el método define una función objetivo estrictamente convexa, coerciva y no restringida. El enfoque provee una caracterización completa de la solución, considerando existencia y unicidad de soluciones y un algoritmo para obtener la solución con buenas condiciones de convergencia. El modelo es aplicable a problemas reales.

Palabras claves: hiper-redes, equilibrio integrado uso de suelo y transporte, modelos logit.

[1] The case with externalities is presented in a twin paper in this Conference.
[2] El caso con externalidades se presenta en un trabajo gemelo en este Congreso.

1. INTRODUCTION

One of the major complexities in modeling big urban areas for planning purposes is to properly represent the strong connection between the transportation system and the spatial distribution of residential and non-residential activities. On the one hand, the activities' spatial patterns represent the major determinant of generation and attraction of trips from(to) each zone, while, on the other hand, the transportation system is also a relevant input for location decisions through the resulting measures of accessibility due to the transportation system design along with the demand conditions for such a system. Therefore, changes in land use and activities directly affect the transportation demand patterns, which can eventually change accessibility, and so on. This multilevel process establishes a complex global equilibrium of the entire urban system, and a close formulation of such a problem can be explored through the interaction between the equilibriums of each subsystem.

In the specialized literature we can find several articles dealing with the interaction between land use and transport (LU&T). Some models postulate the equilibrium in land use taking into consideration the importance of the transport in the location decisions, through a measure of the generalized transportation costs, which are assumed known for the land use equilibrium mechanism. The generalized costs are normally obtained from network assignment models, computed for fixed location patterns. However, from this framework, the authors do not solve simultaneous land use-transport equilibrium. Instead, they find the global equilibrium by means of iterative calculations of partial equilibrium of land use on the one hand, and transport on the other; these schemes are known as bi-level models in the literature. The major concern with regard to these models (apart from the expensive computation involved in solving these problems) is that we can neither ensure existence nor uniqueness of the procedure solution. Therefore, any heuristic method designed for this purpose does not ensure convergence to the global optimum. From all these observations, we can say that to ensure existence as well as uniqueness of LU&T equilibrium is so far an open research topic.

In this paper we develop an integrated LU&T model based on a variational inequality formulation for the equilibrium. A convex optimization problem is defined on an extended network, representing the decisions not only taken at the transport system level but also at the land use system, both in the same graph. We ensure existence as well as uniqueness of the optimum under reasonable assumptions. The first order conditions of this problem reproduce the equilibrium conditions of two previously developed models, the "Random Bidding and Supply Model" (RB&SM) by Martínez and Henríquez (2006) for urban location and the "Markovian Traffic Equilibrium" proposed in Baillon and Cominetti (2006) for private urban transport networks. Thus, the proposed model yields the equilibrium of the integrated LU&T system. One important feature of such models is that all agent decisions can be modeled by a logit model, which also applies in the case of the proposed model for the global equilibrium. Moreover, a solution algorithm is developed, from which its convergence to the global LU&T equilibrium is also proved.

The aforementioned model does consider neither externalities among households nor agglomeration economies among firms. In fact, including these types of interaction does not allow writing the problem as a variational inequality. Nevertheless, we recognize that land use externalities can constitute an important antecedent when taking medium and long term location decisions, for both households and firms. In the case of households, location decisions of certain socioeconomic group members are usually affected by the socioeconomic characteristics of the neighbors, resulting in segregation, among other phenomena. In the case of firms, we observe important interactions among firms that, for example, generate incentives to stay together, called agglomeration economies. Additionally, travelers decide their destinations considering what activities are available at each alternative zone, but also who other travelers are deciding to travel to each zone because it affect social an economic interaction opportunities.

In this paper the lack of modeling externalities is not based on ignoring them, but on assuming that they are lagged in time, so consumers observed past decisions of other households and travelers. Nevertheless, in a twin paper we develop a modified LU&T model, which includes externalities in the locations of the agents. Such a model is formulated as a multidimensional fixed point, and represents the simultaneous equilibrium of land use, generation of trips, distribution and network assignment.

Thus, in Section 3 the approach is presented, conceived to represent the interaction between the land use market and the transport system, written under an integrated formulation: a variational approach for the case without externalities. In the next section, the most relevant previous research efforts to model the interaction between land use and transportation, and the LU&T equilibrium, are briefly described.

2. BACKGROUND

In the specialized literature, we can find several examples of interactions between the land use market and the transportation system (LU&T). According to Chang (2006), the models can be categorized in Spatial Interaction, Mathematical Programming, Random Utility and Bid-Rent models. However, the problem of formulating the equilibrium LU&T is still an open problem, so far described and formulated by using some simplified models (in most cases heuristics), from which there is no analytical way to establish conditions of existence, uniqueness and convergence of such an equilibrium.

The first step to find an integrated formulation LU&T is to properly model the trip structure behind the system. The spatial interaction model proposed by Lowry (1964) and later generalized in Wilson (1970), introduces the concept of cost impedance between zones, explicitly represented by a cost function. Wilson (1970) postulated a model based upon the maximization of the system entropy. Although this case considers a fixed cost between zones, the author introduces a relative measure of the zone attractiveness. The model is not really able to completely explain the relation between land use and transport, mainly due to considering constant transportation costs.

A relevant land use model to discuss is the context of this paper is the RB&SM, which belongs to the “Bid-Rent” type modeling (Martínez and Henríquez, 2006). Real estate transactions are commanded by an auction mechanism, under a best bid rule. In this scheme, the resulting willingness to pay for each location describes the behavior of the decision takers, as proposed by Alonso (1965). The RB&SM model is an extension of the Random Bidding Model (RBM) previously developed by Martínez and Donoso (2001).

Another relevant land use model within the spatial interaction approach, is the doubly constrained entropy model (Roy, 2004), similar to the proposal by Wilson (1970) with the difference that in this case the location is determined by the agents´ willingness to pay assumed to be known. From this model, the logit probabilities proposed by Ellickson (1981) are obtained, because the entropy maximization approach and the multinomial logit are equivalents when the parameters of the latter are estimated with the maximum likelihood method (Anas, 1981). Then, the RB&SM model can also be derived by formulating an optimization problem as a maximization of an entropy function (with neither externalities nor scale economies).

The aforementioned land use models find the equilibrium considering the transport system and its interaction with the land use market. However, in all cases the generalized costs for transport are assumed exogenous.

The trip assignment models determine the route to be followed by each trip once both the mode and destination have been chosen (see Ortúzar and Willumsen, 1994). The process is modeled through the Wardrop network equilibrium conditions, for both determinist and stochastic traffic assignment (Sheffi, 1985). Nagurney and Dong (2002) propose an integrated model and formulates the network assignment problem as a variational inequality, reproducing the Wardrop conditions. Baillon and Cominetti (2006) on the other hand, developed a markovian equilibrium model for stochastic assignment. Unlike the other network equilibrium models (route based decision), the markovian equilibrium models a chain of decisions, where at each node the user decides the next link to get in, pursuing the minimization of the expected travel time to reach a predefined destination, regardless of the assignment decisions taken before.

The link between transport and land use may be identified by access measures, which can be derived from a rigorous microeconomic framework (Martínez, 1995), and has been used to identify the complex relationship between user benefit measures from both the land use and the transport systems (Martínez and Araya, 2000). They explicitly compute the transport benefits associated to origin and destination zones (accessibility and attractiveness) in the context of a doubly constrained spatial interaction model.

One way to deal with the integrated problem is to use mathematical programming in the modeling. That consists on writing the decision problem as an optimization problem. The associated objective function must ensure existence and uniqueness of the global minimum. Chang and Mackett (2005) formulate a bi-level problem to integrate both levels. At the superior level, the location problem is faced under a bid-rent approach by computing the access (accessibility and attractiveness) of the zones. At the inferior level, the network decisions are made taking into account the access measures decided at the higher level. This procedure, however, does not ensure the existence of equilibria. Another model of this type is the one proposed by Boyce y Mattsson (1999) based on mathematical programming, in which the equilibrium at the transport network level, as well as that of land use, are solved through optimization problems. The formulation satisfies equilibrium conditions at the transport level, however, there are no equilibrium conditions attained at the location level.

The objective of this paper is to model the interaction between the transport and the land use system, ensuring existence and uniqueness of equilibrium. Moreover, efficient algorithms to solve these models are proposed, ensuring convergence toward the global optimum of the entire urban system.

Before formulating the model, a glossary of the problem variables is presented:

Exogenous Data

- N : Set of nodes of the transport network.
- D : set of destinations (d) of the trips.
- C : set of nodes for household type (h) searching for a place to be located
- I: set o f zones where households can get a location.
- $\tilde{A}$: set of location arcs ($\tilde{a}$).
- A : Set of links of the transport network.
- A_h^+ : Set of links whose tails are node h.
- A_h^- : Set of links whose heads are node h.
- $s_a(.)$: performance function of link $a \in A$ depending on the traffic flow.
- i_{a_0}, j_{a_0} : tail and head nodes of link a_0, respectively.
- $Q_j^- = \{i \in H \cup N \setminus \exists\, a \in A_i^+, \, j_a = j\}$: set of nodes reachable from node j through a single link.
- $Q_j^+ = \{ j_a \setminus a \in A_j^+ \}$: set of nodes reached from node j through a single link.
- $\tau_i^{jh}(t) = \varphi_i^{jh}(t_a + \tau_{j_a}^{jh}(t)\,; a \in A_i^+)$: Minimum expected time for a trip from the node i to the node j, with the family of functions φ_i^{jh} belonging to the class $\mathcal{E}$ defined in Baillon and Cominetti (2006), *i.e.,* the class of functions which can be expressed in the form $\varphi(x) = E(\min\{\mathrm{x}_1 + \varepsilon_1, ..., \mathrm{x}_\mathrm{n} + \varepsilon_n\})$ where $\varepsilon = (\varepsilon_i)_{i=1}^n$ is a random vector with continuous distribution and $E(\varepsilon) = 0$.
- H_h : total amount of agents of type h.
- S_i : number of real estate supply units at zone i.
- N_h^d : number of trips generated from a household of type h with destination d.
- z_{hi} : willingness to pay component corresponding to the value of location amenities, including accessibility and neighborhood quality given by a household h for a place within zone i.

Steady state variables

- t_a : travel time of link a
- r_i : rent of a real estate located at zone i.
- b_h : monetary disutility index (bid) for type agent h.
- H_{hi} : number of households located at location i at equilibrium. H is the matrix of households locations.

3. INTEGRATED LAND USE – TRANSPORT MODEL WITHOUT EXTERNALITIES: BASE MODEL (FIXED SUPPLY)

In the markovian equilibrium scheme (MTE) by Baillon and Cominetti (2006), the authors develop a traffic flow assignment model on a transportation network, assuming known the trip distribution patterns. They search for the equilibrium by means of the minimization of an objective function defined on the transport network. In this first formulation we extend the MTE model, but considering a more complex network, in which fictitious arcs and nodes are added in order to represent the agents' location decision. Then, the original objective function is modified to represent the LU&T integrated problem.

In Figure 1 the extended proposed network *G(N,A)* is depicted. In this scheme a household *h* choose the optimal path including an initial arc from a fictitious node *h* to the location node *i*. This node also belongs to the network from which the members of household *h* will start their trips. The trip generation process is represented by a second layer in the network. Each trip chooses an optimal path through the transport network and finishes at a destination node *d*. We assume that the number of trips generated is constant for each household type, and also that the destination in the transport network is decided by the optimal path; only one transport mode is assumed in the formulation framework.

The cost assigned to the location arcs represents the willingness to pay of the households for the land use around the end (head) node of such an arc. The location is decided through a bid-rent mechanism in the location decisions. Under this approach, the willingness to pay function represents the agents' behavior in the location decisions (Alonso, 1965). In this model, the following willingness to pay function is postulated:

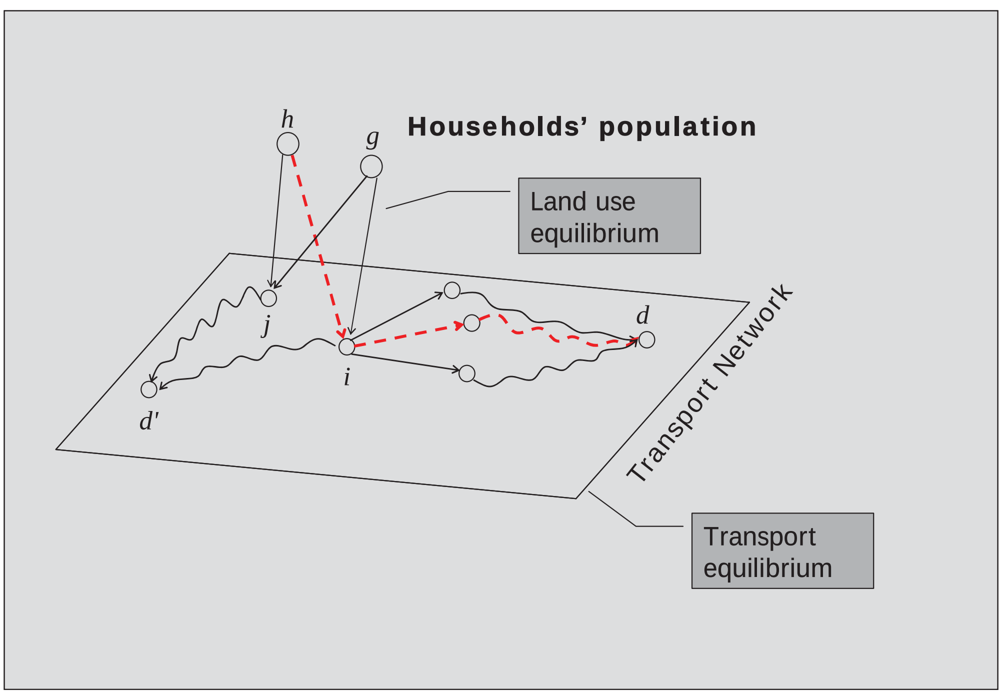

Figure 1: Urban system equilibrium: Hyper-network representation

$$B_{hi}(b_h, t) = -b_h + z_{hi} - \sum_{d \in D} N_h^d \tau_i^d(t) \tag{1}$$

Note that this form for the willingness to pay is derived from assuming: a quasi-linear subjacent utility function, in at least one of the consumption goods; an exogenous household income; and that the consumer chooses only one location for residence. It can be shown that under these assumptions $b_h = y_h - u_h / \eta_h$, where y_h is the income, u_h the utility level and η_h the marginal utility of income (Martínez and Henríquez, 2006). Thus, the first term shows the utility level reached by a household of type *h*, which has to be the same for all households of the same type under equilibrium conditions. The second term is assumed to be constant and it captures how a household of type *h* values the attributes of a specific zone *i*, in this case assuming that location externalities are exogenous to the equilibrium process. Finally, the third term provides a measure for the transport cost, as explained ahead in this paper, assuming only one trip purpose and N_h^d trips per each destination zone *d* exogenously given.

The values of state variables at the optimum are found by solving the following optimization problem:

$$\min_{t,r,b} \Phi(t,r,b) = \sum_{a\in A} \int_0^{t_a} s_a^{-1}(z)dz + \sum_{h\in C} H_h b_h + \sum_{i\in I} S_i r_i + \frac{1}{\mu} \sum_{\substack{h\in C \\ i\in I}} e^{\mu\left(B_{hi}(b_h,t)-r_i\right)} \tag{2}$$

This problem may be explained as the dual of the following doubly constrained maximum entropy problem (P), assuming transport costs as fixed:

$$(P) \qquad \min_{\mathrm{H_{hi}}} \; -\sum_{\substack{i\in I \\ h\in C}} H_{hi} Z_{hi}(t) + \frac{1}{\mu} \sum_{\substack{i\in I \\ h\in C}} H_{hi}[\ln(H_{hi})-1]$$

$$\text{s.a.} \qquad \sum_{i\in I} H_{hi} = H_h \qquad \forall \;\; h \in C$$

$$\sum_{h\in C} H_{hi} = S_i \qquad \forall \;\; i \in I$$

where $Z_{hi}(t) = z_{hi} - \sum_{d\in D} N_h^d \tau_i^d(t)$. This problem conceptually represents the maximization of bids under stochastic bids, or the representation of the auction prices.

In this model, the function $\Phi : R^{|A|+|I|+|C|} \to R$, where R is the real set. Moreover, the number of households located at each zone i, is bounded from above by the zone supply, which is assumed to be fixed in this scheme. Additionally, equation (2) is invariant to shifts in *b* and *r*, that is $\min \Phi(t,r,b) = \min \Phi(t, r-c, b+c) \quad \forall c \in R$. Like in the markovian equilibrium model, the objective function in (2) is strictly convex and coercive, under the assumptions (Hyp A), $b_1 = 0$ and $\sum_{i\in I} S_i = \sum_{h\in C} H_h$. These conditions ensure both existence and uniqueness of the solution at the optimum.

(Hyp A)= { *The family* $\varphi_i^d \in \varepsilon$, *with* $\varphi_d^d \equiv 0$; *travel time functions* $s_a : R_+ \to R$ *are strictly increasing continuous, with* $\lim_{x\to\infty} s_a(x) = \infty$ *and* $s_a(0) = t_a^0 \geq 0$; *and* $\varphi_i^d(t^0) > 0$ *for all* $i \neq d$

The first order conditions of problem (2) characterize the equilibrium, not only in the bid-rent market (RB&SM) but also in the transport system, through the traffic markovian equilibrium MTE. Analytically, the first optimality condition is

$$\forall h \in C \quad \frac{\partial \Phi}{\partial b_h}(t,b,r) = 0 \quad \Rightarrow \quad H_h = \sum_{i \in I} \exp(\mu(B_{hi}(b_h,t) - r_i)) \tag{3}$$

Defining $H_{hi} := \exp(\mu(B_{hi}(b_h,t)-r_i))$ that corresponds to the total number of type h families located at zone i, the following land-use equilibrium condition is fulfilled:

$$H_h = \sum_{i \in I} H_{hi} \qquad \forall h \in C \tag{4}$$

Moreover, by replacing the expression for $B_{hi}(b_h,t)$ in (3), we obtain:

$$\exp(-\mu b_h) = \frac{H_h}{\sum_{i \in I} \exp(\mu(z_{hi} - \sum_{d \in D} N_h^d \tau_i^d(t) - r_i))} \tag{5}$$

Thus, replacing (3) in the definition of H_{hi} yields:

$$H_{hi} = H_h \cdot \frac{\exp(\mu(Z_{hi}(t)-r_i))}{\sum_{j \in I} \exp(\mu(Z_{hj}(t)-r_j))} = H_h \cdot \frac{\exp(\mu(B_{hi}(b_h,t)-r_i))}{\sum_{j \in I} \exp(\mu(B_{hj}(b_h,t)-r_j))} = H_h \cdot P_{i/h} \tag{6}$$

where $P_{i/h}$ represents the probability for a household of type h to prefer a real estate at zone i. Note that Martínez (1992b) obtains this probability (which is called "choice"), assuming that, on the one hand, the household' surplus, defined by $\Delta_{hi} = B_{hi}(b_h,t) - r_i$, distributes Gumbel identical and independent (*iid*) with dispersion parameter μ, and on the other hand, the household choose in order to maximize such a surplus.

From equation (3) we can also deduce that, for each $h \in C$, (13) applies:

$$b_h = \frac{1}{\mu} \ln\left(\sum_{i \in I} \exp(\mu(Z_{hi}(t)-r_i)) \right) - \frac{1}{\mu} \ln(H_h), \tag{7}$$

Note that equation (7) reproduces fixed point problem for the RB&SM market clearance equilibrium condition.

A second optimality condition is

$$\forall i \in I, \quad \frac{\partial \Phi}{\partial r_i}(t,b,r) = 0 \Leftrightarrow S_i = \sum_{h \in C} \exp(\mu(B_{hi}(b_h,t) - r_i)) \tag{8}$$

which is equivalent to $S_i = \sum_{h \in H} H_{hi}$. From this relation, we can obtain:

$$\exp(-\mu \cdot r_i) = \frac{S_i}{\sum_{i \in I} \exp(\mu \cdot B_{hi}(b_h,t))} \qquad \forall i \in I \tag{9}$$

and
$$H_{hi} = S_i \cdot \frac{\exp(\mu \cdot B_{hi}(b_h,t))}{\sum_{g \in C} \exp(\mu \cdot B_{gi}(b_g,t))} = S_i \cdot P_{h/i} \tag{10}$$

where $P_{h/i}$ is called the "bid" probability in the RB&SM. This represents the probability that the consumer of type *h* is the best bidder in location *i* competing with all bidders in *C*, which is derived in Martínez (1992) by assuming bids distributed *iid* Gumbel with parameter μ .
Moreover, from (10) we obtain that, for each $i \in I$:

$$r_i = \frac{1}{\mu} \ln\left(\sum_{h \in C} \exp(\mu \cdot B_{hi}(b_h,t)) \right) - \frac{1}{\mu} \ln(S_i), \tag{11}$$

Expression (11) is, again, the same as the result obtained in the RB&SM model, and can be interpreted as the expected value of the maximum expected willingness to pay among the households asking for a place to be located.
In addition, a third condition is provided by the equilibrium on the transport network, from where the following first order conditions are obtained:

$$\forall a \in A, \quad \frac{\partial \Phi}{\partial t_a}(t,b,r) = s_a^{-1}(t_a) + \frac{1}{\mu} \sum_{\substack{h \in C \\ i \in I}} \exp(\mu(B_{hi}(b_h,t) - r_i)) \cdot \mu \frac{\partial B_{hi}}{\partial t_a}(b_h,t),$$

where
$$\frac{\partial B_{hi}}{\partial t_a}(b_h,t) = -\sum_{d \in D} N_h^d \cdot \frac{\partial \tau_i^d}{\partial t_a}(t).$$

Then,

$$\frac{\partial \Phi}{\partial t_a}(t,b,r) = 0 \quad \Rightarrow \quad s_a^{-1}(t_a) = \sum_{d \in D} \sum_{i \in I} \left(\sum_{h \in C} H_{hi} N_h^d \right) \frac{\partial \tau_i^d}{\partial t_a}(t)$$

By defining $g_i^d := \sum_{h \in C} H_{hi} N_h^d$, representing the total number of trips whose destination is d starting at zone i , we obtain:

$$s_a^{-1}(\mathrm{t_a}) = \sum_{d \in D} \sum_{i \in I} g_i^d \frac{\partial \tau_i^d}{\partial t_a}(t) = \sum_{d \in D} \langle g^d, \frac{\partial \tau^d}{\partial t_a}(t) \rangle = \sum_{d \in D} v_a^d = w_a, \tag{12}$$

from where, the equilibrium condition on the arcs belonging to the transport network system is reproduced, $t_a = s_a(w_a)$.

From such results, we can prove that the solution of the optimization problem in (2) simultaneously satisfies the equilibrium conditions of both models, RB&SM and MTE.

Remark

Several extensions of the proposed model can be analytically developed. One straightforward extension is to add the distribution of trips problem into the integrated modeling framework. Another interesting extension is to incorporate the real estate supply as an endogenous variable. For further details see Briceño (2006).

4. SIMULATIONS

The purpose of the simulations is to analyze the prediction performance of the model. The solution algorithm uses a global MSA (successive averages method), which is a gradient-type method to find the equilibrium on traffic flows, with an internal fixed point algorithm to find the location equilibrium at each iteration.

Two basic scenarios are generated and the equilibria is obtained for each one. Locations externalities were considered as lagged interactions, where consumers take into account the past distribution of residents and there choices are sensitive to segregation effects between households of different types, i.e. category dislikes living next (in the same zone) to some others (representing the rich class), while the second one likes living close to the first one (representing the poor class). The function that represents these externalities is linear.

The algorithm was implemented in MATLAB over the known Siouxfalls network, which has 24 nodes and 76 arcs, with a travel time function of the form $s_a(w_a) = t_a^0[1 + b_a(w_a / c_a)^{p_a}]$. This function satisfies the conditions given before in order to assure the existence and uniqueness of the equilibrium. For arc choice in the transport network we considered a logit model with a scale parameter β independent of the node, household cluster, destination or trip purpose. That is:

$$\tau_i^{dph}(t) = -\frac{1}{\beta} \ln \sum_{a \in A_i^+} \exp(-\beta(t_a + \tau_{j_a}^{dph}(t)))$$

The basic scenario (named 0) considers: 5 categories of households divided into two groups: the poor class includes categories 1 to 3 and the rich class includes categories 4 and 5; only 1 trip purpose available in the whole set of nodes; high attraction factors on 5 special nodes of the network which makes that almost all network trips are attracted to this neighborhood, called neighborhood A; inelastic population of consumers (H_h) and supply of locations (S_i), initially spread homogeneously over the network's nodes. Besides, through constants z_{hi} we simulated the preference of the poor class for living in neighborhood A, while the rich class prefer neighborhood B represented by four neighbor nodes. Thus, neighborhood A represents a poor residential area but also an employment district, while neighborhood B is a high class residential neighborhood.

We designed simulation that shows subsequent scenarios divided into two sets. The first set, denominated k^{-}-*scenarios*, shows the influence of the differentiated amenities z_{hi}, then the second set, denominated k^{+}-*scenarios (k>0)*, shows the influence of increasing congestion created by increasing demand H_h in all categories. In the first set, k^{-}scenarios (*k<0), z_{hi}* increases linearly according to $z_{hi}^{k} = z_{hi}^{0}(1-k/4)$, thus, for example, the influence of z_{hi} in the -4 scenario is null. In the second set, k^{+}-*scenarios*, demand and supply increases in successive k^{+}-scenarios as follows: $H_h^k = H_h^0 \cdot 2^k$ and $S_i^k = S_i^0 \cdot 2^k$.

Figure 3 assumes that neighborhoods attract residents regarding the location of households in the previous iteration, but in a way that segregation between poor and rich is inherent in their behavior. It depicts the share of households' classes (rich in red and poor in blue) that locates in neighborhood A at equilibrium, and its evolution along different scenarios. In the first set of scenarios *(k<0)*, it can be seen that as the preference for the neighborhood A increases, represented by the increase in the respective z_{hi} in successive scenarios, the poor class tends to concentrate in this neighborhood while the rich class concentrates in neighborhood B. This shows the different preference for neighborhoods between socioeconomic classes. In the second set of scenarios (*k>0)*, the increase in congestion make more attractive for the rich class to live next to their jobs, which explains the tendency of this class to outbid the poor in neighborhood B. In this case, the rich class faces two opposite preference: the preference for neighborhood B and the reduced travel costs by living in A.

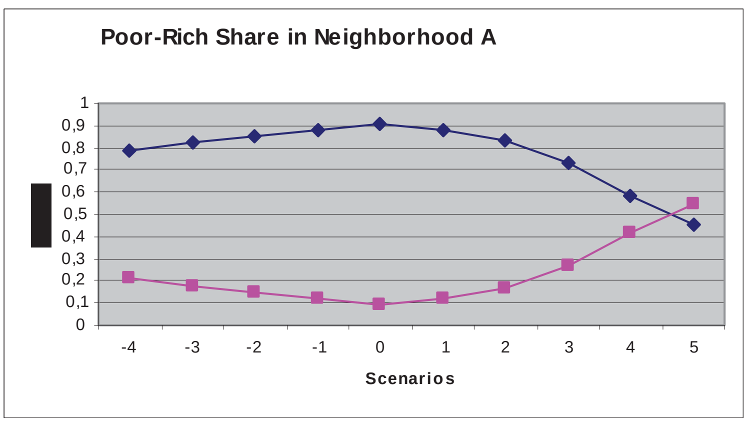

Figure 2: Location dynamics without externalities

The model attains equilibrium in 24 seconds on a 3.2GHz PC.

5. CONCLUSIONS AND FURTHER RESEARCH

The model developed here allows the full integration of the land use and the transportation systems in the case without externalities, based on hyper-networks, i.e. an extension of the classical transport network to represent the land use market equilibrium. The model follows the classical Beckmans' approach of defining an equivalent optimization problem that reproduces Wardrop's traffic equilibrium conditions. The contribution of our model is to extend the approach to simultaneously reproduce the transport and land use equilibria under a unique optimization problem. The main theoretical result is the proof of a unique solution for the LU&T system. The model can be used in real contexts by assuming that externalities are lagged in one or more periods, i.e. consumers make choices using information of the land use system that takes time to be acquired. In this sense, this model is regarded as a partial equilibrium model. One limitation of the model is the treatment of the transport system as only private transport modes. The extension to public transport assignment requires further research.

The hyper-network model of the urban system can be seen as a platform for modeling other dimensions of the urban system. Further developments may include the information and the goods markets, as additional layers in the hyper-network.

Finally, the hyper-network approach can be used to specify dynamic urban processes on the hyper-network, including equilibrium stages along time on each submarket, in line with Martínez and Hurtubia (2006). This would allow considering delays in infrastructure development and the introduction of lack of information on key variables of decision makers, like on expected future prices. The equilibrium problem so far developed provides the basic structure for such further extensions.

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