A THEORETICAL TIME HIERARQUICAL MICROECONOMIC MODEL OF ACTIVITIES: IMPLICATIONS INTHEVALUE OFTIMEANDSUB-OPTIMAL DECISIONS

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Abstract

The microeconomic approach to explain the consumers' behavior regarding consumption of goods, choice of activities and the use of time is extended in this paper by including explicitly the temporal dimension on making choices. Here, some choices involve a long term commitment, like jobs or education, and other are made and modified in the short term, like leisure, shopping and transport. The temporal scale defines the time window to adjustactivities (duration and location),well as the magnitude of the resources (time and money) spent. We specify and analyze a stylized model with two time scales: the macro and micro levels, concluding that the observation of preferences, therefore the value of time,at a micro level, like transport mode choice, are strongly conditioned by the prevailing choices in the macro scale.

Keywords: microeconomic and mathematical models, time allocation of activities, Hierarchical and sub-optimal decisions, value of time

1. INTRODUCTION

In recent years, various microeconomic and mathematical models have been designed on explain people's behavior, either on the consumption of discrete or continues goods, the choice of feasible activities within a system or the use of time for them. Becker(1965),presentsa microeconomic modelbased on theincorporationoftime allocatedtodifferent activities inthe satisfaction or utility of eachindividual.Heassumes that theindividualdoes not incorporate inthetimeallocated to work in the utility function. In this context, the authorconcludesthatthevalueoftimeequalstheconsumer's wagerate. De Serpa (1971) introduces the time as a second argument in the utility function and addsa set of technological relationships between consumption goods and duration time for each activity, whereas the consumption of a specific amount of any good requires a minimum amount of time. This model yields an expression for the time value as a resource allocated to any activity. In addition, it introduces the concept of leisure activities as those which the consumer would like to increase the duration or time and or frequency.By explicitly modeling the choice of transport mode as a component of the activities, Train and McFadden (1978) derived the specification ofthe indirect utility function for the mode choice, whose parameters yield values of time, derived in this case from the consumer's choice between the alternative time and cost options offered by different transport modes.LaterJara-Díaz and Guevara (2003) and Jara-Díaz and Guerra (2003) related the choicesof activities and transport modes.

The aboveresearch focused on the value of time has in common the assumption that all choices are optimized instantaneously. In this paper we explore the implications of relaxing this assumption introducing the time scale of activities under the otherwise same approach. Our argument is that activities performed by an individual can be clearly differentiated by the duration of timewindow that the activity last, and that such time window is directly related with the proportion of available resources (time and income) consumed or produced by the activity. For example, a choice of work and study involve many years of commitment and consumes a large proportion of daily time and the monetary resources, while other activities, like shopping or leisure choices, can be modified within a day or week and involve of lower amount of resources. We consider the consumer's problem that optimizes utility by choosing a set of activities, i.e. described by both time duration and goods consumption, with different frequencies including fast and slow variables which interest on time. This feature is common in many physical and a biological system, which have provided the knowledge of their common dynamics structures and has been recently applied to study social and economic system (see Gunderson and Holling, 2002). Dynamic systems are usually modeled by assuming a hierarchy in the processes involved, with different time scales for each sub-system, which is the approach followed in this paper to model the individuals activities. There are other features common to dynamic systems, like stochastic shocks and memory effects, as well as conditions on the decisions sequence that represent a potentially strong non-linearity in the dynamic process. Despite their relevance in our problem they are not included in our model for the benefit of simplicity and comparability with the previous research on the value of time; we seek to isolate the relevance of the temporal scale in the activities decision process.

The model explicitly considers ahierarchical structure, based on thetime windows or the frequency on which choices are made, to represent the speed of change and the amount of resources consumed. We study the case of two temporalscales, named as macro and micro scales, arguing that this simple version is enough to understand the effect that replicates in a multi-scales structure. Using this macro-microstructure we seek to analyze the dynamic that exists in the individual's behavior and, particularly, to study the effects that random shocks in the economy and long term choices may have on the set of decisions taken at the micro level, as well ason the interpretation of the value of time when is derived from choices at the micro scale.We proposethe followinginteractionsinthemicroeconomicmodelwithtwoscalesofdecision: The micro scale choice of activities is conditioned by the choices made at the macro scale, because the latter cannot be adjusted as frequently. Therefore, at the micro scale the set of feasible activities depends on the durables, goods and services, available as a result of choices made at the macro scale. Finally, the resources, time and wealth, available to decide on micro scaleactivities is constrained by what is left after deducting the expenditure on macro activities.An important assumption of the proposed modeling is the timing or synchronyin making decisions at the macro and micro levels. That is, the beginning of each macro-scale, all short-term decisions are adjustable at this instant of time called the adjusting point. We assume that the individual makes decisions based on the current economic information on each given time window (myopic decision). This assumption only affects exogenous parametersin the model, and is easily expandable or modified.

Inaddition, we showthat the value of time associated to micro scale activities is dependent on the set of choices made at the macro scale, including expenditure in durables. Then the value of time estimated from the econometric calibration of transport mode choice models is bound to be different according to the individual's choice of housing, car(s), education level, marriage status, job, as well as the location of macro scale activities, like work and study. This macro-micro direction of dependency among activities' choices is not explicit in classic models where all activities belong to the same hierarchy level.

There are empirical evidencesupporting the argument thatlong-term decisionsinfluence the allocationandvaluationof time in the short-term activities For example see, Levinson(1999), BullardandFeigenbaum(2007), Guang-Zhen and Yew-Kwang, (2009),Yáñez (2010). In our model we explain theoretically such results by defining a hierarchy in the set of individual's decisions with consequences in the dynamics of the allocation of time.

2. MICROECONOMIC HIERARCHICAL MODEL OF CONSUMER

The proposed model considers the hierarchy of activities depending of the flexibility or frequency of changes in consumption of both continuousand discrete goods over a specific horizon of time. To analyze this hierarchy, the feasible space of activity choices is divided intwo scales, called macro scale (long-term activities) and micro scale (short-term activities). The macro scalechoices are exemplified by work and study, which after defining their basic contract conditions (salary or fees, duration and location) they remain essentially fixed during the macro period of time. The micro scale includes decisions that can be adjusted frequently (e.g. daily), exemplified by leisure andshopping. The individual's choices on the location of activities are assumed to be decided at the same timescale corresponding to the activity, but decisions regarding transport mode and route choices are taken in the micro scale, irrespective of the activity to which the consumer is travelling.

Activities are defined as self-contained, meaning that activities do not share any action, goods or time allocations between them, and interdependent, meaning that activities may need resources acquired on other activities to be feasible.At each temporal scale consumers make choices assuming that all exogenous variables, like prices, are constant during the time window; hence the longer the time window the more risky the price prediction the consumer has to make. It is also assumed that some goods obtained in the macro scale are durable in nature, are defined as goods (or services) that once acquired can be used many times over time, also known as reusable assets (Sullivan and Stevens, 2003). In addition, their characteristics affect the level of consumption at the micro scale.The model developed in this paper is deterministic, because we assume that the individual knows the state of the economy (model parameters) at each moment of time where decisions are made.

2.1 Notation

All activities, macro or micro, and their consumption goods and durations, are defined in a common "time unit", irrespective of the temporal scale.

Relevant sets and indices:

- T:Planning horizon.
- Δt, ΔT =V Δt: Duration of time window at the micro-scale, e.g. a day or a week ,and duration of time window at the macro-scale, e.g. one to five years. V is the number of micro time windows in the macro window, and N is number of macro-scale time windows during the planning horizon, $T = N\Delta T$.
- J: Sets of time scales. In the two scales model J={j_1=m, j_2=M}, with m denoting micro and M denoting macro scales. In addition, Ω^j, Λ^j are the sets of feasible activities/ consumption goods associated with scale j.$\Omega^M \cap \Omega^m = \emptyset; \Lambda^M \cap \Lambda^m = \emptyset$.
- *i, k, n, v*: Index for activities, consumption goods, time windows of the macro scale, time windows of the micro scales.

Parameters:

- $\boldsymbol{y_i^{n,v}}$, $\boldsymbol{c_i^{n,v}}$Income and cost per time unit obtained from *micro* activity i in the micro time window v, that belongs to the macro time window n *(n,v)*, $v = 1, \dots, V;\ n = 1, \dots, N;\ \forall i \in \Omega^m$.
- $\boldsymbol{p_k^{n,v} P_k^{n,v}}$**:** Unitary price of good k(micro and macro, respectively) in *(n,v)*; $\forall k \in \Lambda^m U \Lambda^M$
- $\boldsymbol{Y_i^{n,v}}, \boldsymbol{C_i^{n,v}}$**:** Income and Cost per time unit obtained from MACRO activity i in the period n,v.$\forall i \in \Omega^M$
- $\boldsymbol{I^{n,v}}$: Exogenous income in (n,v) obtained through of real estate rents/Cost per time unit, investment in previous periods, etc.

Decision Variables:

The classical decision variables are the allocation of time and the consumption of goods in each activity. The new variables in this model are the time and wealth surpluses, which are transferred from the macro scale to the micro scale; these are called control variables.

Decision variables in the direct utility specification:

- $X_k^{n,v}, x_k^{n,v}$: Consumption of good k (macro or micro, respectively) k –per unit of time Δt – that belongs to the micro time window n,v;$\forall k \in \Lambda^M U \Lambda^m$
- $T_i^{n,v}, t_i^{n,v}$: Time allocated to activity i (macro or micro, respectively) in(n,v); $i \in \Omega^M U \Omega^m$.

Control Variables (Macro-micro transference):

- $\boldsymbol{S_M^{n,v}}$: Surplus of wealth –per time unit–, which is transferred from the macro decisions (time and consumption) in period (n,v) to the micro time window v ($\boldsymbol{S^{n,v}} \in \boldsymbol{R}$) with $\boldsymbol{S^{n,v}}$ a debt (negative) or a saving (positive).
- $\boldsymbol{\tau^n}$: Time saved from macro window n into activities at the micro scale.
- $\boldsymbol{s_m^{n,v}}$: Surplus of wealth –per time unit–, which is transferred from the micro window n,v to the next micro time window $(n, v+1)$ if $v < V$ or $(n+1,1)$ if $v = V$;$\boldsymbol{s^{n,v} \geq 0}$.

2.2. Time Constraints

Within each time window, following Jara-Díaz (2003), we assume that the decision of time assigned to certain activity depends on the level of consumption of goods. In the macro scale, the decision depends on the consumption goods at thatscale only, but at the microscale it depends on the choices of consumption at the macro as well as the micro scale. This assumption reflects that some micro scale activities require inputs decided in the long term (infrastructure, durables). For example, a car is a durable that is an input to choose this mode on every activity; housing is another durable whose type (floor space, land lot size and building quality) affects micro scale choices like leisure.

Let us define the following constraints associated with the macro scale:

$g_i^n(T_i^{n,v}, X^{n,v}) \geq 0, \ \ \forall i \in \Omega^M$: The time and goods allocated to activity i within a time window are dependent. We consider particularly the two different forms proposed by De Serpa (1971) and extended by Jara-Díaz (2003):

- $T_i^{n,v} \leq \overline{D}_{,i}^n(X^{n,v}), \ \ \forall i \in \Omega^M$: The maximum feasible time for the realization of activity i is constrained by the quantity of consumption goods purchased on thesame macro scale time window n.
- $T_i^{n,v} \geq \underline{D}_{,i}^n(X^{n,v}), \ \ \forall i \in \Omega^M$: The minimum time for an activity is constrained by set of consumption goods on the same macro scale time window n.

Note that, the macro technological constraints are fixed for the entire time window *n*.
With regard to the micro scale, we identify the following types of constraints,$g_i^{n,v}(t_i^{n,v}, x^{n,v}; X^{n,v}) \geq 0, \ \ \forall i \in \Omega^m$: these constraints relate the time assigned to activity i at the micro time scale,with both micro and macro consumption decisions. As described for the macro case, this constraint can be, for example:

$$\underline{d}_i^{n,v}(x^{n,v}, X^{n,v}) \leq t_i^{n,v} \leq \overline{d}_i^{n,v}(x^{n,v}, X^{n,v})$$

That is,bothin thespace ofmicro andmacrodecisions, thetimeallocated toan activity maybejointlybounded from aboveandfrom below bytheresources obtainedorendogenousand exogenous constraints.

2.3. Optimization Model

Let us first define the activity choice and consumption problem of the consumerduring a micro time window v associated with the macro time window n (n,v).

In a first step, wedescribe the formulation of macro problem (long term decisions). Such decisions are taken at the beginning of a macro window, ie at time (n, 1) and are fixed for all v belonging to micro n. Analytically the formulation of macro problem byfor any time window (n, v) is described in (1). In the hierarchical formulation, this problem is solved only in *v = 1*.

$$\max_{X,T} U_M^{n,v}(X_k, T_i, i \in \Omega^M, k \in \Lambda^M)$$

Subject to

$$F_M^{n,v} \equiv \begin{Bmatrix} \sum_{i \in \Omega^M} T_i^{n,v}(Y_i^{n,v} - C_i^{n,v}) - \sum_{k \in \Lambda^M} P_k^{n,v} X_k^{n,v} + I^{n,v} - S_M^{n,v} = 0 \\ \sum_{i \in \Omega^M} T_i^{n,v} = \Delta t - \tau^n \\ \underline{D}_i^n(X^{n,v}) \leq T_i^{n,v} \leq \overline{D}_i^n(X^{n,v}), \ \ \forall i \in \Omega^M, \end{Bmatrix} (1)$$

Thelength of the decision time unit is assumed equal to Δt.

After solving the allocation of time and consumption of goods at the macro level, the consumer has the possibility to make choices at the micro level, for every the following V micro time windows, conditional on the choices made at the macro level. Themresources constrained at the micro level, by the macro choices andsurpluses $S^{n,v}$and τ^{n}, where the time period nmust cover the micro time window v. Note that the micro choice problem cannot be solved for all micro level windows v at the beginning of the macro period because of the uncertainty about the variation of external conditions (such as prices p or congestion levels) along the macro period. Therefore, during the time horizon T, the macro choice problem is solved *N* times and the micro choiceproblem is solved$T = V \times N$ times. We define the following problem of the consumer's decision at the micro level v time window, conditional on the consumption goodsdecided within the macro window n of macro problem (1):

$$\max_{x,t} U_m^{n,v}(x_k, t_i, i \in \Omega^m, k \in \Lambda^m)$$

Subject to

$$F_m \equiv \begin{Bmatrix} \sum_{i\in\Omega^m} t_i^{n,v}\left(y_i^{n,v} - c_i^{n,v}\right) - \sum_{k\in\Lambda^m} p_k^{n,v} x_k^{n,v} + S_M^{n,v} + s_m^{n,v-1} \geq 0 \\ \sum_{i\in\Omega^m} t_i^{n,v} = \tau^n \\ \underline{d}_i^{n,v}(x^{n,v}, X^{n,v}) \leq t_i^{n,v} \leq \overline{d}_i^{n,v}(x^{n,v}, X^{n,v}), \;\; \forall i \in \Omega^m \end{Bmatrix} \quad (2)$$

For simplicity, weassume in the micro problem thatthe interest rateforsavings $s_m^{n,v-1}$iszero.

Afterfindingthe optimal solution of$t_i^{n,v}$ and $x_k^{n,v}$notedas$t_i^{n,v*}$ and $x_k^{n,v*}$for themicroproblemin (n, v) is calculated$s_m^{n,v}$ as thesurplus oftheincome constraint,then

$$s_m^{n,v} = \sum_{i\in\Omega^m} t_i^{n,v*}\left(y_i^{n,v} - c_i^{n,v}\right) - \sum_{k\in\Lambda^m} p_k^{n,v} x_k^{n,v*} + S_M^{n,v} + s_m^{n,v-1} \geq 0 \qquad (3)$$

3.**Hierarchical dynamic of decisions**

In this section, we show the process of making decisions based on a hierarchicalstructure of decisions.When an individual ends a macro time window, she/he change in long-term decisions, there is anadjusting point. In this moment, theindividual obtains the time assigned to different activities and goods consumption (macro and micro together). In addition, the adjusting point between the macro time windows n-1 and n is assumed fixed and the individual solves the following optimization problem (simultaneous problem in n,1).

$$\max_{x,T,x,t} \left(\alpha U_M^{n,1}\left(X_k^{n,1}, T_i^{n,1}, i \in \Omega^M, k \in \Lambda^M\right) + (1-\alpha) U_m^{n,1}\left(x_k^{n,1}, t_i^{n,1}, i \in \Omega^m, k \in \Lambda^m\right) \right)$$

$$\text{subject to:} \qquad F_m^{n,1} \cup F_M^{n,1} \qquad (4)$$

The set$F_m^{n,1} \cup F_M^{n,1}$isthesimultaneousmicro-macro feasible set in(n,1). That is, theindividualmakes macro decisions based(amongotherthings)intheirchoicesat the micro levelatthat instantof time (n,1). For example, the decisions of residential activitiesare based intheenvironmental featuresthatallow theagent to makesmicroactivities(shopping, social activities, interaction withotheragents,etc.). In addition, the consumer chooses his/her activities, time and goods within a set of possibilities generated by the Pareto frontier obtained by parametrically varying parameter α between [0,1]. If $\alpha \to 1$, represents the behavior of a person valuing the future and if $\alpha \to 0$, someone that does not plan anything in advance

We note the economic and constraint parameters as:

$$\varepsilon_M^{n,v} \equiv (Y^{n,v} - C^{n,v}, P^{n,1}, I^{n,v}, \theta_M^{n,v}), \qquad (5)$$

$$\varepsilon_m^{n,v} \equiv (y^{n,v} - c^{n,v}, p^{n,v}, s^{n,v-1}, \theta_m^{n,v}), \qquad (6)$$

Then

$$X_k^{n,1*}(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta t), \ \forall k \in \Lambda^M, \quad T_i^{n,1*}(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta t), \ \forall i \in \Omega^M, \quad (7)$$

In addition, the macro decisions are fixed in the macro time window n, them:

$$X_k^{n,v*} = X_k^{n,1*}, \ T_i^{n,v*} = T_i^{n,1*}; \quad \forall k \in \Lambda^M, \forall i \in \Omega^m; v = 1, \dots, V \quad (8)$$

Note that the long-term decisions are taken at time period*(n, 1),*andthese choices affect all macro time window n.

After that an individual determines long-term commitments (durable goods, fixed-time activities), its possible define a surplus or transfer money (cost / revenue committed) and transfer time for short-term decisions. These amounts of time and money decisions are transferred from the macro (macro scale) into the macro window n tomake short-term decisions in each time window micro v (v ≠ 1) as

$$S_M^{n,v} = \sum_{i \in \Omega^M} (Y_i^{n,v} - C_i^{n,v}) T_i^{n,1*} - \sum_{k \in \Lambda^M} P_k^{n,v} X_k^{n,1*} + I^{n,v} \quad (9)$$

$$\tau^n = \Delta t - \sum_{i \in \Omega^M} T_i^{n,1*} \quad (10)$$

The value of $S_M^{n,v}$ can change always *v ≠ 1 micro time windows because the income, cost, prices, rents can change in the time, them* $S_M^{n,v_1} \neq S_M^{n,v_2}$ with $v_1 \neq v_2$. However, τ^n is constants for all v. Mathematically, the expression that explains the variation is

$$S^{n,v}(\varepsilon_M^{n,v} | \varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta \mathrm{t}, \theta^{n,1}) \quad (11)$$

On the other hand, we have the following decision process at the micro level. After solving the optimization macro problem (4) in the adjusting point (n, 1), the individual makes short-term decisions at (n, v) in each micro time window *v ≠ 1*, that are valid for a period Δ*t,* that is, solving the micro problem (2), assuming that the macro decisions are fixed (consumption and time), so that the solution to this problem is:

$$x_k^{n,v}(\varepsilon_m^{n,v}, S_M^{n,v}, \tau^n, X^n, \theta_m^{n,v}), \ \forall k \in \Lambda^m, \qquad t_i^{n,v}(\varepsilon_m^{n,v}, S_M^{n,v}, \tau^n, X^n, \theta_m^{n,v}), \ \forall i \in \Omega^m \quad (12)$$

We definethemacro indirectutility functioninthe microtime window(n, v) as:

$$V_M^{n,v} = \alpha U_M^{n,v}(X_k^{n,1*}, T_i^{n,1*}, \forall k \in \Lambda^M, \forall i \in \Omega^M) \quad (13)$$

Therefore, $V_M^{n,v} \equiv V_M^{n,v}(Y^{n,1} - C^{n,1}, y^{n,1} - c^{n,1}, P^{n,1}, p^{n,1}, I^{n,1}, \Delta t, \theta^{n,1})$

Orequivalently$V_M^{n,v} \equiv V_M^{n,v}(\varepsilon_M^{n,1}, \varepsilon_m^{n,1}, \Delta t, \theta^{n,1})$(14)

Similarly, we define the micro indirect utility function for the micro time window (n, v>1) as

$$V_m^{n,v} \equiv (1 - \alpha) U_m^{n,v}(x_k^{n,v*}, t_i^{n,v*}; \forall k \in \Lambda^m, \forall i \in \Omega^m) \quad (15)$$

Therefore,

$$V_m^{n,v} \equiv V_m^{n,v}(\varepsilon_m^{n,v}, S_M^{n,v}, \tau^n, X^n, \theta_m^{n,v}) (16)$$

Additionally (16) satisfies the following properties of monotony in $S_M^{n,v}$ and τ^n

- If $S_{M,1}^{n,v} \leq S_{M,2}^{n,v}$and $0 < \tau^n \leq \Delta t$, then
  $$V_m^{n,v}(\varepsilon_m^{n,v}, S_{M,1}^{n,v}, \tau^n, X^n, \theta_m^{n,v}) \leq V_m^{n,v}(\varepsilon_m^{n,v}, S_{M,2}^{n,v}, \tau^n, X^n, \theta_m^{n,v}) \quad (17),$$
  For a fixed value ofτ^n .
- Given τ_1^n, τ_2^n such that $\tau_1^n \leq \tau_2^n \leq \Delta t$ then

$$V_m^{n,v}(\varepsilon_m^{n,v}, S_M^{n,v}, \tau_1^n, X^n, \theta_m^{n,1}) \le V_m^{n,v}(\varepsilon_m^{n,v}, S_M^{n,v}, \tau_2^n, X^n, \theta_m^{n,1}), \quad (18)$$

This is valid for a fixed value of $S_M^{n,v}$.

This result is important because it is a support to analyze the sub-optimality in the transfer of macro-micro resources and the sub-optimality of long-term and short-term decisions.

4.ANALYSIS OF VALUE OF TIME

In a first step, we analyze the expression for the value of time,following Jara-Díaz 2003 and DeSerpa, 1971 in each micro time window (n,v) to analyze the dynamics of such measure and the influence of the macro decisions.

Let us define$\lambda^{n,v}$, $\mu^{n,v}$, $\kappa_{i(-)}^{n,v}$, $\kappa_{i(+)}^{n,v}$ as the Lagrange multipliers associated with constraints of income, time and the minimum or maximum time allocated, all with respect to micro activity i in time window (n,v).We can calculate the value of time in each point in time in the dynamic processwhere we identify the following two characteristic points: micro time window at any *point v>1* (the general case) and at the adjusting point *v=1* (the adjusting point).

4.1 **General case micro time window (n,v) v≠1**

Assume that the value of $\lambda^{n,v}$ is nonzero, ieindividualsdo not savemoney between micro time windows. The first order conditions for the micro good consumption$k \in \Lambda^m$ in the time window (n,v) are:

$$(1-\alpha)\frac{\frac{\partial U_m^{n,v}}{\partial x_k^{n,v}}}{\lambda^{n,v}} - p_k^{n,v} + \sum_{i\in\Omega^m}\left(\frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}}\frac{\partial d_{max,i}^{n,v}(x^{n,v}, X^n)}{\partial x_k^{n,v}} - \frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}}\frac{\partial d_{min,i}^{n,v}(x^{n,v}, X^n)}{\partial x_k^{n,v}}\right) = 0, \quad \forall k \in \Lambda^m \qquad (19)$$

The equation (19) indicates that the consumption of goods at the micro level is conditional on the consumption decisions at the macro level. Macro decisions are found at the beginning of the respective macro window and are fixed in the micro time window (n,v), v≠1. Thevalue of time (20) is:

$$\frac{\mu^{n,v}}{\lambda^{n,v}} = (1-\alpha)\frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} + (y_i^{n,v} - c_i^{n,v}) + \frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}} - \frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}}; \forall i \in \Omega^m; v = 2, \dots, V \qquad (20)$$

The value of time within a micro time window (n,v), depends on the value of time allocated to activity i, the net income of the activity, and a factor that depends on the feasibility of allocating time given the set goods (decided at the macro and micro levels) available for the consumer.
In addition, the solution of thesetofequationsof themicro problemin(n, v) yields the optimal allocation of consumer goods and time allocated to micro activitiesconditionalonX^n, τ^n and exogenousparameters.Because we assumeendogenoustechnological constraintsordifferent constraintsdepending on the typeof agent thatmakes macro decisions, then, the values of $\kappa_{i(-)}^{n,v}$, $\kappa_{i(+)}^{n,v}$ may vary according to specific characteristics of the agent (e.g. minimum travel time according to the residential location and job location or minimum time to work). Thus, we concludethatis possible to findindividualswithidentical conditions (same wage rate $y_i^n - c_i^n$and the same valuationoftheactivity$\frac{(1-\alpha)}{\lambda^{n,v}}\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}$) but with differences in the value of time as a resource.

Note that from complementary slackness conditions on the technological constraints we have:

$$\kappa_{i(-)}^{n,v}\left(t_i^{n,v} - d_{min,i}^{n,v}(x^{n,v}, X^n)\right) = 0, \qquad \kappa_{i(+)}^{n,v}\left(d_{max,i}^{n,v}(x^{n,v}, X^n) - t_i^{n,v}\right) = 0, \qquad (21)$$

$$and\ \kappa_{i(+)}^{n,v} \times \kappa_{i(-)}^{n,v} = 0, \forall i \in \Omega^m$$

The conditions of (21) can be analyzed in the following 3 sub-cases:

- **SUB-CASE 1.1:** $\kappa_{i(+)}^{n,v} = 0 \; and \; \kappa_{i(-)}^{n,v}$=0:

The time allocated to activity i is strictly bounded by the upper and lower constraints. More specifically the following are true:

Upper bound constraint: the optimum complies with$t_i^{n,v} < d_{max,i}^{n,v}(x^{n,v}, X^n)$, which means that durables goods are enough in to accomplish the activity optimally. In other words, there is a surplus in consumption of macro level goods such that the time spent in the activity can be increased.

Lower bound constraint: at the optimum$t_i^{n,v} > d_{min,i}^{n,v}(x^{n,v}, X^n)$, which means that the consumer spends more time than the minimum to perform activity i.

- **SUB-CASE 1.2:**$t_i^{n,v} - d_{min,i}^{n,v}(x^{n,v}, X^n) = 0, \;\; \kappa_{i(+)}^{n,v} = 0, \;\; \kappa_{i(-)}^{n,v} > 0$

That is, the lower bound constraint for the time allocated to activity i is saturated in the micro time window (n, v) indicating that the individual assigned the minimum time possible.Inthisway, we obtain an expression (22)for the individual's marginal willingness to pay for reducing the time spent in activity *i*

$$\frac{\kappa_{i(-)}^{n,v}}{\lambda^{n,v}} = \frac{\mu^{n,v}}{\lambda^{n,v}} - (1-\alpha)\frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} - \left(y_i^{n,v} - c_i^{n,v}\right); \qquad (22)$$

This result is equivalent to that found by DeSerpa (1971) This willingness to pay for reducing the time allocated to this activity is equal to the value of time in the respective micro window minus the marginal valuation to allocate time for the activity minus the income (plus the cost) generated per unit time for the respective activity.

- **SUB-CASE 1.3:**$d_{max,i}^{n,v}(x^{n,v}, X^n) - t_i^{n,v} = 0, \;\; \kappa_{i(+)}^{n,v} > 0, \;\; \kappa_{i(-)}^{n,v} = 0.$

In this case the upper bound constraint is active or saturated, $d_{max,i}^{n,v}(x^{n,v}, X^n) \geq t_i^{n,v}$then

$$\frac{\kappa_{i(+)}^{n,v}}{\lambda^{n,v}} = (1-\alpha)\frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} + \left(y_i^{n,v} - c_i^{n,v}\right) - \frac{\mu^{n,v}}{\lambda^{n,v}}, \qquad (23)$$

which represents the individual´s marginal willingness to pay for increasing the time spent in activity i above the maximum capacity. In this case, an individual is willing to pay to increase the time allocated an activity i the following value:the valuation associated to make and assign time to the activity, more the income per unit of time (minus costs), minus the value of time as a resource in micro time window (n, v).Assuming that $y_i^{n,v} - c_i^{n,v} \leq 0$, then he/she has the intention to pay for increasing the time of such activity if

$$(1-\alpha)\frac{\frac{\partial U_m^{n,v}}{\partial t_i^{n,v}}}{\lambda^{n,v}} \geq \left(c_i^{n,v} - y_i^{n,v}\right) + \frac{\mu^{n,v}}{\lambda^{n,v}},$$

That is, the marginal valuation of the individual's time allocated to activity is higher than the cost per unit time plus the time value as a resource for micro window (n, v). The equation above shows an extension of classical models on the definition of the value of leisure, for example, Jara-Díaz (2003) based in the work of De Serpa (1971) classifies all activities with $\kappa_i^{n,v} = 0$ as leisure, because the author considers lower bounds only, then he concludes that extra time allocated is only justified by the pleasure it brings to the consumer. This is justified in the context of mono time scale because it is assumed that goods in the economy are not constrained for the individual. Although in our model this conclusion holds, it is only valid for $\kappa_i^{n,v} = 0$

associated to lower bounds. If the multiplier is associated to an upper bound it is not necessarily leisure, it can be unpleasant but providing extra resources that the individual would like to increase is the bound were relaxed. This is the case of working time which cannot be increased because decisions taken in the macro scale prevents it, like a working contract.

4.2. Case ofadjusting point

If$K_{i(-)}^{n,1}$ and $K_{i(+)}^{n,1}$are the Lagrange multipliers associated with constraint the minimum or maximum time allocated to (macro or micro) activity i in time window *(n,1).*

$$\frac{\mu^{n,1}}{\lambda^{n,1}} = \frac{\frac{\partial U^{n,1}}{\partial T_i^{n,1}}}{\lambda^{n,1}} + \left(Y_i^{n,1} - C_i^{n,1}\right) + \frac{K_{i(-)}^{n,1}}{\lambda^{n,1}} - \frac{K_{i(+)}^{n,1}}{\lambda^{n,1}}; \forall i \in \Omega^M U \Omega^M, \qquad (26)$$

For other hand, the willingness to pay for reducing the time allocated to a macro activity in the adjusting time (n,1) is defined as:

$$\frac{K_{i(-)}^{n,1}}{\lambda^{n,1}} = \frac{\mu^{n,1}}{\lambda^{n,1}} - \alpha \frac{\left.\frac{\partial U_M^{n,1}}{\partial T_i^{n,1}}\right|_{T^{1*},X^{1*}}}{\lambda^{n,1}} - \left(Y_i^{n,1} - C_i^{n,1}\right), \qquad (27)$$

As long-term decisions are not re-optimized in the micro time window v> 1, then the willingness to pay for reducing the time of the macro activity i in the time window v can be calculated (approximately)using the marginal rates and the parameters of the micro problem in (n, v) and the equation (27):

$$WTPD_i^{n,v} = \frac{\mu^{n,v}}{\lambda^{n,v}} - \alpha \frac{\left.\frac{\partial U_M^{n,v}}{\partial T_i^{n,v}}\right|_{T^{1*},X^{1*}}}{\lambda^{n,v}} - \left(Y_i^{n,v} - C_i^{n,v}\right) \quad (28)$$

Where, the value of time as resource for the micro time window *(n,v)* can also be defined as (v>1).

$$\frac{\mu^{n,v}}{\lambda^{n,v}} = \frac{\frac{\partial V_m^{n,v}}{\partial \tau^{n,v}}}{\frac{\partial V_m^{n,v}}{\partial S_M^{n,v}}}; \qquad (29)$$

Equivalently, it is possible to findanexpressionthatapproximates the willingness to pay for increasing the time of macro leisure activity (long term activity) in the micro time window *(n,v), v>1.*

5. HIERARQUICAL DECISIONS VS SIMULTANEOUS DECISIONS

In this section, we analyze the impact of long-term decisions on short-term decisions and the sub-optimality in the value of time that can be obtained by the lack of more frequently adjustment of the long-term decisions. Wecalledsimultaneousmicroeconomic problem in (n,v) to the classic static consumer problem, equivalent to (Jara-Diaz, 2003). Using the notation of this paper, the formulation of simultaneous microeconomic problem in (n,v) is:

$$\max_{x,T,x,t} \left(\alpha U_M^{n,v}\left(X_k^{n,v}, T_i^{n,v}, i \in \Omega^M, k \in \Lambda^M\right) + (1-\alpha) U_m^{n,v}\left(x_k^{n,v}, t_i^{n,v}, i \in \Omega^m, k \in \Lambda^m\right)\right)$$

$$F_m^{n,v} \cup F_M^{n,v} \qquad (30)$$

The indirect utility function for the simultaneous problem in the time window n,vis:

$$V_{simu}^{n,v} \equiv V_{simu}^{n,v}(\varepsilon^{n,v}, \Delta t, \theta^{n,v}), \qquad (31)$$

Where$\varepsilon^{v} \equiv (Y^{v} - C^{v}, P^{v}, I^{v} + s_{m}^{v-1})$.

If$v > 1$ and

$$V_{simu}^{n,v}(\varepsilon^{n,v}, \Delta t, \theta^{n,v}) > \alpha V_{M}^{n,v}\left(\varepsilon_{M}^{n,1}, \varepsilon_{m}^{n,1}, \Delta t, \theta^{n,1}\right) + (1-\alpha) V_{m}^{n,v}(\varepsilon_{m}^{n,v}, S_{M}^{n,v}, \tau^{n}, X^{n}, \theta_{m}^{n,v}),$$

Then the individual is outside of its optimal solution for the structure of the hierarchy problem. Thus, we may define transfers of money and time in the simultaneousproblem, such as income and time generated by micro decisions, or equivalently, the transfer of money (32) and time(33) of the macro decisions to short-term decisions.

$$S_{sim}^{n,v} \equiv (Y_{M}^{n,v} - C_{M}^{n,v}) T_{sim}^{n,v*} - P_{M}^{n,v} X_{sim}^{n,v*} + I^{n,v}, \quad (32)$$

$$\tau_{sim}^{n,v} \equiv \Delta t - \sum_{i \in \Omega^{M}} T_{sim,i}^{n,v*}, \qquad (33)$$

Where, $T_{sim}^{n,v*}$ and $X_{sim}^{n,v*}$ are the solutions of the macro variables in the simultaneous problem.

By the structure of the simultaneousproblem, there may be differences in the values of money and time transfer of the macro to the micro decisions ($S_{sim}^{v}(\varepsilon_{M}^{v}, \varepsilon_{m}^{v}, \Delta t, \theta^{v})$ and$\tau_{sim}^{v}(\varepsilon_{M}^{v}, \varepsilon_{m}^{v}, \Delta t, \theta^{v})$) with respect to the hierarchy transferences, ($S_{M}^{n,v}(\varepsilon_{M}^{1}, \varepsilon_{m}^{1}, \Delta \mathrm{t}, \theta^{1})$ and $\tau^{1}(\varepsilon_{M}^{1}, \varepsilon_{m}^{1}, \Delta \mathrm{t}, \theta^{1})$) .

Thesolutions of micro variables in the simultaneous problemare the same to

$$\max_{\mathrm{x,t}} (1-\alpha) U_{m}^{v}(x^{v}, t^{v})$$

$$F_{m,sim}^{n,v} \equiv \left\{ \begin{array}{c} (y_{m}^{n,v} - c_{m}^{n,v}) t^{n,v} - p_{m}^{n,v} x^{n,v} + S_{sim}^{n,v} + s^{v-1} \geq 0 \quad (\lambda_{sim}^{n,v}) \\ \sum_{i \in \Omega^{\mathrm{m}}} t_{i}^{n,v} = \tau_{sim}^{n,v} \qquad (\mu_{sim}^{n,v}) \\ d_{min,i}^{n,v}\left(x^{n,v}, X_{sim}^{n,v}\right) \leq t_{i}^{n,v} \leq d_{max,i}^{n,v}\left(x^{v}, X_{sim}^{n,v}\right), \ \forall i \in \Omega^{m} \end{array} \right\} \qquad (34)$$

Obtained that the value of time in the classical problem of consumer (simultaneous problem) is the marginal rate of substitution between time and money

$$\frac{\mu_{sim}^{n,v}}{\lambda_{sim}^{n,v}} = \frac{\frac{\partial V_{m}^{n,vv}}{\partial \tau_{sim}^{n,v}}}{\frac{\partial V_{m}^{n,v}}{\partial S_{sim}^{n,v}}}, \quad (35)$$

This formulation is important to compare the differences in the valuation of time obtained through the constraints endogenous by generated by the long-term decisions, with the model that includes flexibility in these decisions (simultaneous problem). Below, there are two cases where the valuation of time of the integrated problem is different with respect to the valuation of time of hierarchical modeling. Assuming that $n = 1, and\ s_{m}^{v} = 0$, for all v.

Case 1 (Exogenousincome**):**

We suppose that anindividualassumeslong-term commitments$X$, and exogenousincomedefinedas $I^{v} \equiv Y^{v} T_{work}^{min}$, where Y^{v} is the wage rate and T_{work}^{min} is the fixed timeallocatedtowork.Additionally, allshort-term decisions generates cost ($y_{i}^{v} - c_{i}^{v} < 0$, for all $i \in \Omega^{m}$), then

$$\sum_{i\in\Omega^m}(y_i^v - c_i^v)t_i^v - \sum_{k\in\Lambda^m} p_k^v x_k^v < 0.$$

Theonlysourceofincomeisthe job activitythroughafixed income. Then $S_M^v = I^v - P^v X^v$ and $\tau^v = \Delta t - T_{work}^{min}$. If$P^v$increaseswith respectto adjusting point(P^1) then$X_{int}^v \leq X^1$and$P^v X_{int}^v \leq P^v X^1$, therefore$S_{sim}^v \geq S_M^vandT_{sim,work}^v \geq T_{work}^v$then$\tau_{sim}^v \leq \tau^1$.That is, the individual does not have the optimum macro-transfer of money for micro activities and micro consumption in the time window v (v> 1) and resources may not be enough. On the other hand, the marginal utilities of the micro problem$\lambda^v(S)$ and $\mu^v(\tau)$ in the time window $v > 1$, are decreasing functions, obtaining that $\lambda^v(S_{sim}^v) \leq \lambda^v(S_M^v)$or $\mu^v(\tau_{sim}^v) \geq \mu^v(\tau^1)$. This implies that

$$VST_{int}^v = \frac{\mu^v(\tau_{sim}^v)}{\lambda^v(S_{sim}^v)} \geq \frac{\mu^v(\tau^1)}{\lambda^v(S_M^v)} = VST^v$$

the individual is willing to pay more to have more time for micro activities with respect to the valuation obtained through the hierarchical structure of decision.

Case 2(Endogenousincome)**:**

Assumingthatanindividual haslong-term commitmentsassociated withtimeallocatedto workT^v,butthe wage ratevaries forv> 1. If$\frac{\partial U}{\partial T^v} < 0$andthewage rateincreases,thenthetime allocatedto workwill be less in a simultaneous problem(if$T^v > T^{min}$). In this case,$S_M^v = I^v T^v$, $\tau^v = \Delta t - T^v$. If I^v increases with respect to(I^1) then $T_{sim}^v \leq T^v$whereby $S_{int}^v \leq S_M^v$ and $\tau_{sim}^v \geq \tau^1$. That is, the individual does not have the optimum transfer of money, because he/she is working longer than optimal.

$$VST_{sim}^{n,v} = \frac{\mu^{n,v}(\tau_{sim}^v)}{\lambda^{n,v}(S_{sim}^v)} \leq \frac{\mu^{n,v}(\tau^1)}{\lambda^{n,v}(S_M^v)} = VST^v$$

Then the value of time of simultaneous problem is overestimated using the micro value of time. In this way, we define two gaps of sub-optimality in the value of time

Micro-macro gap in the micro time window v:

$$\Delta VT_h^{n,v} = \left|\frac{\mu^{n,1}}{\lambda^{n,1}} - \frac{\mu^{n,v}}{\lambda^{n,v}}\right|$$

This measure analyzes the dynamics (variation) of the value of time in each micro time window after of the adjusting point.

Micro-long run gap in the micro time window n, v

$$\Delta VT_{sim,h}^{n,v} = \left|\frac{\mu_{sim}^{n,v}}{\lambda_{sim}^{n,v}} - \frac{\mu^{n,v}}{\lambda^{n,v}}\right|,$$

This measure analyzes the sub-optimality of hierarchical value of time in each micro time window (n,v)

Note that in each adjusting point, the gaps are equal to zero due to the fact that thesimultaneous solution equals the hierarchical solution. It is worth noting that in this paper, we assume that the adjusting point isknownandexogenous. Withoutlossof generality the model can be formulated with endogenous adjusting pointsdependingona decision maker’s criteria, which is a matter for further research.

Numerical Example

In the followingnumerical example, we assume that the only inter-temporal variation is the wage rate. The dynamics of the wage rate assumed in this example is shown in Figure N. 1:

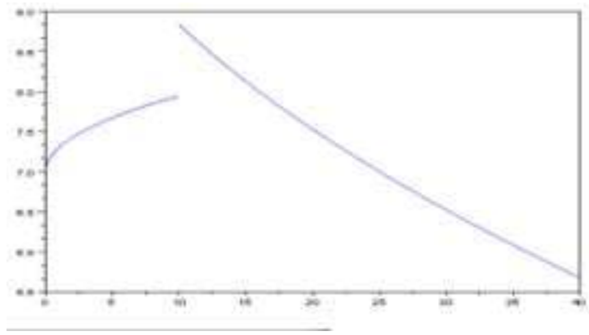

Figure 1: Changesinthe wage rate(measured inmonetary units) with respect tothedecision stage y^{v}_{work}

In the next Figure 2, we show the variation of the value of time, of two hierarchical problems with adjusting point v=9 (VT (HP-9)) and v=15 (VT(HP-15)), which can be compared with the simultaneous problem values of time (VT(PI))

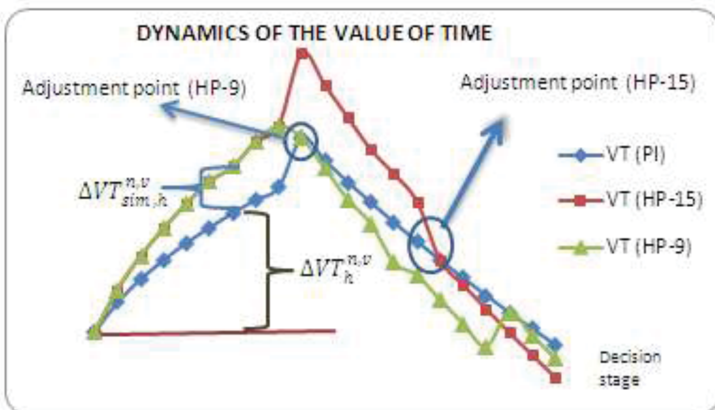

Figure 2: Changethe values oftime (hierarchicalandsimultaneousproblems)

The dynamics of the value of time (VT) in the simultaneous problem (PI) is similar to the dynamics of the wage rate, because this value is the only source of change. Before the first adjusting point (for both cases) hierarchical values of time (HP cases) are greater than thesimultaneous value of time (PI case), becausein the latter case, in each period the individual reduces the number of hours assigned to work as the wage rate increases, hence the income reduces and so does the value of time; conversely, in the HP case longer working hours remain fixed (at the value decided in v=1) until the next adjusting point.

CONCLUSIONS

The paper presents an extension of the classical theory of allocation of time to consider a standard feature observed in several dynamic systems: the temporal hierarchy of processes. Here model developed considers two levels, the macro and micro temporal levels, to describe the process of the individual's allocation of time and wealth on activities. No other dynamic feature was included, while except for this extension the model setting is identical to the classical one level approach.

Despite the simplicity of the model, our analysis yields some relevant conclusions that contribute to the theory of the value and allocation of time.The classic and the new approaches are consistent, because both collapse to the same model once we assume that all choices are taken and adjusted simultaneously, that is, there is no hierarchy in the process of deciding activities. Therefore the hierarchical model is a generalization of the former.

We do not identify relevant shortcomings caused from choosing only two levels. In fact, from our analysis we maintain the claim that the simple micro-micro framework is the nucleus of more complex hierarchies, in the sense that it can represent the most relevant effects of moving from one to multiple levels.

The most general conclusion is that the theory of the allocation of time is enriched by the hierarchical structure of activities; conversely, the assumption of one temporal level obscures some features that explain the consumers' behavior with relevant implications in modeling travel demand and time allocation.

The main contribution of the hierarchical approach is that it makes clear how the behavior at the micro level is conditional on the decisions made at the macro level. This has impact in the methodologies to estimate demand and on the understanding of the formation of the value of time. Some shortcomings of the classical one level model has been partially overcome by clustering the population with regards to some long term decisions,like car ownership, gender, residential location and income level, based on intuitive arguments. Our model gives theoretical support to such intuition, but it also provides a theoretical framework to identify a complete set of variables to define the population clusters.

More importantly, our results show that clustering can be avoided, at least partially, by specifying the indirect utility function of choices at the micro explicitly dependent on the macro level choices. The benefit this approach is the reduction of parameters necessary in the demand model, which reduces the number of observation required to obtain a given precision on the parameter estimate. A model with less number of parameters with the same precision is always a better forecasting model.

The theory of the value and allocation of time is complemented by the model. It explains how macro level decision and their value of time depends on the expected effect on other decisions at the macro and micro levels, while decisions at the micro level are dependent on the resources allocated on activities engaged at the macro level.Thus, the estimate of the value of time inferred from the consumer's behavior depends on the type of activity and whether it is decided at a macro or micro level.

An important extension to this paper refers to the model calibration. To achieve this objective it isnecessary to obtain orgenerate adynamic databaseon the samegroup of individuals, i.e. a panel data,and analyzethe allocation of timein different temporary cohorts,incorporating the effect ofeconomic variables(income, investments, rents), demographic (locationhome, work,etc.) and changesin the cycleof life (income, age, work, etc.). With this information,it is possible to estimate thevalue of timein each period based on the classiceconometrics for static models.

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