A Quasi-equilibrium Logit Model for Residential Suburbanization

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Abstract: In the suburban context, where commuting trips from suburbs to towns is significant, the assumption of land market equilibrium conditions for residential and non-residential allocation is partially relaxed. We allow dynamic effects like delays in dissemination of information of the state of endogenous variables, e.g. rents and consumers interactions. Additionally, consumers are assumed myopic agents, i.e. they cannot forecast the city's dynamics, while suppliers can only forecast the aggregate population but not their demand for each location. In this setting, a quasi-equilibrium set of conditions is fulfilled at each point in time based on the bid-auction theory. These quasi-equilibrium conditions are implemented in a logit-auction model, which simplifies the calculation of the solution compared with the equilibrium equivalent model; however, it increases the number of periods where quasi-equilibrium has to be calculated (every one or two years). Distinct features of this model are path independency, endogenously calculated rents as the result of auctions, excess of supply and/or demand at any point in time for each location and a unique solution only dependent on the initial conditions. The model is applied to study the expected relocation of residents and firms following a rail project for commuters, interacting the land use model with a four steps transport model.

Keywords: quasi-equilibrium, logit, auction, land use, commuting

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1. INTRODUCTION

There is a long lasting debate, mostly in conferences, regarding the use of market equilibrium approach in modeling urban markets. The large number and the complexity of equilibrium conditions fuel this debate, including: the classical market clearing supply-equal-demand condition that yields equilibrium prices; the number of interactions social between consumers known as location externalities among residents and economic interactions among firms known as agglomeration economies; the competition between suppliers themselves facing economies of scale; and the interaction between land and transport markets. Additional difficulties to calculate equilibrium arise from discontinuities: locations and real estate are discrete and differentiable by their distinctive attributes. This setting configures a highly complex system that is the theoretical matter of urban economics; see, for example, Alonso (1964) for a classical paper, Fujita et al. (1999) for an advanced theory and Arnott and McMillen (2006) for a compendium of issues in this field.

In the applied field, researchers have divert into two modeling approaches, producing what is named as equilibrium and simulation models, although most of them use the logit model to estimate optimal choices assuming rational agents facing discrete location options. Equilibrium models also follow two approaches: McFadden's maximum utility framework (McFadden, 1977; Anas, 1982) and Alonso's bid-auction framework (Ellickson, 1981); they subsequently converged into a unified bid-choice approach (Martínez, 1992). These models define different sets of equilibrium conditions, although they all find equilibrium prices for land and/or real estate. Simulation models, on the other hand, define rules for the allocation of agents at every point in time. Consumers and available locations are randomly chosen from the respective sets of potential locators and available locations. In this approach all interactions are delayed in time. A gross comparison between these two methodologies shows that: while equilibrium models yield unique prices and path independent allocations (under mild conditions), at the cost of solving complex non-linear problems, simulation models are simple, usually linear problems, easy to implement but require a larger amount of parameters to be calibrated and the solution on prices and allocation is path dependent.

In this paper we intend to contribute in closing the gap between these two approaches, extending the concept of equilibrium to one of quasi-equilibrium. In contrast to the disequilibrium approach used in simulation models, we define market quasi-equilibrium as a system's state fulfilling a set of relaxed equilibrium conditions. This set of quasi-equilibrium conditions assumes that market-clearing prices do not hold, although market forces push prices towards equilibrium.

We assume that information takes times to disseminate among agents (residents, firms and real estate developers). This implies that interactions between agents in the location choice process are lagged by one period. These interactions are: i) between land-dwellings consumers and real estate suppliers; ii) between location choice and changes in transport accessibility; and iii) on consumers' evaluation of land use changes to assess neighborhood quality. The assumption of time lags for information to disseminate also affects how reservation utilities are calculated. This implies that consumers cannot obtain information of other agents' willingness to pay instantaneously (at the same period of the location choice), so they use the information of willingness to pay values revealed in the past period. Moreover, we also assume that consumers adjust their reservation utilities at each point in time, following the Random Bid and Supply Model (RB&SM) developed by Martínez and Henríquez (2007). However, in the quasi-equilibrium approach we assume that all consumers have lagged information about other consumers' reservation utilities. The effect of information lags is that the market is not guaranteed to clear at each period and for each location option; this comment is valid even if the total real estate supply matches total demand in the city. This occurs because each location alternative (v,i) is differentiable, so they may have excess of demand or supply for each specific alternative even though the city total supply matches total household units.

A second crucial assumption is that consumers are myopic, which means that consumers cannot anticipate the systems dynamics, i.e. rents and land use, in order to optimize their location choice. Here, we notice that there is an alternative theoretical argument that points out the fact that rational consumers may be able to anticipate the system dynamics, called the perfect foresight assumption (see for example Anas and Arnott, 1991). To illustrate this argument assumes that a single family unit is able to forecast the land use and rents at every location and at each point in the time horizon. This, we think, is a theoretically elegant construct but clearly unrealistic as a valid assumption for applied models; conversely, we argue that for atomized households units making choices in the complex urban context, with high spatial and agents' diversity, the myopic assumption is more realistic. A third assumption is that suppliers are also myopic, but they can forecast a single figure of demography: the total number of agents including households and firms.

Considering information delays and myopic behavior, the quasi-equilibrium model estimates land prices and allocation of consumers by way of the maximum bidder's rule at each location, internalizing all delays. The model yields stable outputs for each time period, with unique solutions for rents land use, conditional on the transport and demography evolution along time and the periods' length.

The model was designed and applied for suburban contexts to forecast the location dynamics of residential and non-residential activities and passengers' trips. By suburban we refer to an area with a large city, hereafter called the city, surrounded by neighbor villages, towns or smaller cities, called towns, such that significant commuting trips between the city and the towns take place. This setting defines an open land market, including urban and interstitial rural areas, all potentially usable for residents and firms. Migration takes place between the city and towns, and urbanized areas can also expand to rural land. The special layout of the city and towns is not relevant as any one can be modeled with our method as long as they are represented in a zoning system. The essential point is the existence of commuting, which establishes the dependency between the city and towns' social and economic activities.

In this paper we present a resumed version of the model concentrated in residential location, although the model was developed integrating residential and non-residential markets. The behavior of firms and their allocation is similar to the residential model herein presented, although in the non-residential the allocation may be defined for firm units, jobs or floor-space units using the same methodology.

In the next Section 2 we present the model following by the estimation of relevant parameters; in Section 3 we develop the quasi-equilibrium conditions and the calculation algorithm is described; in Section 4 we present the application to the Valparaíso-La Calera (Central Chile) corridor where the model was used to assess the train extension.

2. THE LAND USE MODEL

The quasi-equilibrium model can be resumed as a simulation of static states that evolve from one period to another, fueled by a combination of external shocks and internal lagged interactions forces pushing the system towards equilibrium. External shocks are changes in population, in the economy and on public policies, while endogenous interactions are location externalities, land and real estate prices, real estate supply adjustments and changes in commuting conditions.

At each time period *t*, the model updates external shocks: changes in the number and type of households and firms in the city; changes in zoning regulations; and policies on transport and land use markets. It also updates lagged effects on accessibility or transport costs, and on the neighborhood quality attributes or location externalities. Then consumers' willingness to pay values are calculated for the set of real estate options available, based on a reservation utility and on the available information of location attributes (access and zone quality). An absentee auctioneer assigns real estate options to best bidder applying a logit model and reveals market prices. In this step producers decide what to offer at each location maximizing the profit based on the lagged information of land use and prices. The building process delays supply in one period to reach the market. Additionally, supplies forecast the city aggregate demand for housing and produce to match this amount.

Bid-auction assignment of real estates and rents

The population of residents is divided into socioeconomic clusters (firms into types of business) indexed by $h=1,\ldots,N_h$. The spatial context is partitioned into discrete zones denoted by index $i=1,\ldots,N_i$. A taxonomy of the real estate types divide supply options denoted by index $v=1,\ldots,N_v$.

The model estimates the number of households cluster *h* that is allocated to a real estate options *(v,i)* at period *t*, denoted H_{hvi}^{t}, which distributes the real estate units across population. This allocation is calculated by:

$$H_{hvi}^{t} = V_{vi}^{t} \cdot P_{h/vi}^{t} \ , \quad \forall h, v, i \tag{1}$$

with V_{vi}^{t} the number of real estate units type (*v,i*) in period *t* and $P_{h/vi}^{t}$ the bid-auction probability that an agent from cluster *h* is the best bidder in a real estate option (*v,i*) in period *t*. This probability is obtained from maximizing the random bids or willingness to pay across the set of locating agents.

To derive random bids we consider the following random (indirect) utility function:

$$\bar{U}_{hvi}^{t} = U_{h}^{t}\left(y_h - r_{vi}^{t-1}, d_{hv}, z_i^{t-1}\right) + \varepsilon_{hvi}^{t} \quad \forall h, v, i, t \tag{2}$$

where U_{hvi}^{t} is the systematic utility components of the random utility, which assess the following attributes' vectors: neighborhood quality and accessibility indices $z_i=(z_{ik},k=1,\ldots,Z)$ and dwelling and socioeconomic indices $d_{hi}=(d_{hvk},k=1,\ldots D)$. Additionally, indirect utility depends on income minus lagged rent $(y_h - r_{vi}^{t-1})$. In the model, income and socioeconomic characteristics define residents' and firms' clusters, then they are fixed variables (parameters) for all periods; similarly real estate attributes define the typology for real estates so they are also fixed parameters. Contrarily, zone attributes, accessibility and rents are endogenous lagged variables under the assumption of delayed information discussed above. In the application below utility functions are assumed linear in parameters, and parameters are differentiated by consumers cluster.

The consumer's willingness to pay can be derived directly from a quasi-linear utility function, which is characterized to be linear in the term $y_h - r_{vi}$; we illustrate the derivation using the following linear utility function:

$$\bar{U}_{hvi}^t = \lambda_h (y_h^t - r_{vi}^{t-1}) + \beta_h \cdot z_i^{t-1} + \gamma_h \cdot d_{hk} + \varepsilon_{hvi}^t \quad \forall h, v, i, t \qquad (3)$$

This can be interpreted as the maximum utility attainable by the consumer at rent the value *r.* Then, replace the rent in Equation 3 by the corresponding willingness to pay (also called bid), denoted by B_{hvi}^t, which represents the maximum value that *h* can pay to attain a utility level *U*. Next, solve Equation 3 for *B* to obtain:

$$\bar{B}_{hvi}^t = b_h^t + B_{hvi}^t + {\varepsilon'}_{hvi}^t \quad \forall h, v, i, t \qquad (4)$$

with $B_{hvi}^t(z) = {\beta'}_h \cdot z_i^{t-1} + {\gamma'}_h \cdot d_{hk}$ and $b_h^t = y_h - U_h^t / \lambda_h$. The bid B_{hvi}^t represents the maximum value the household *h* is willing to pay for a real estate (*v,i*) in order to obtain a reservation utility U_h^t defined by b_h^t .

All parameters are the original utility parameters divided by the marginal utility of income, i.e. $\beta' = \beta / \lambda_h$. The parameter b_h^t deserves special attention because, in contrast with other variables in the bid function, it represents a latent (unobserved) utility level, which in this model is estimated endogenously by imposing a set of quasi-equilibrium conditions.

An important assumption in the model is that the random term of bids, ${\varepsilon'}_{hvi} = \varepsilon_{hvi} / \lambda_h$, is assumed distributed Gumbel with parameter μ, under this assumption the support space for bids is the real set $\mathcal{R}$. The interpretation for negative bids is that to achieve a given utility level, the consumer willingness to pay to compensate a very unattractive location may be negative (i.e. needs a monetary compensation); a real example of this case is the case of NIMBY (not in my backyard) residuals.

From the above it follows that the probability that a given agent *h* is the maximum bidder in option (*v,i*) is given by the following bid-auction probability:

$$p_{h/vi}^t \equiv \frac{H_h^t \exp \mu (b_h^t + B_{hvi}^t)}{\sum_g H_g^t \exp \mu (b_g^t + B_{gvi}^t)} \quad \forall h, v, i, t \qquad (5)$$

where the clusters' total number of households H_h^t is used to account for the known McFadden (1977)'s size correction.

The random bid-auction model also yields rents as the maximum expected bids at each real estate auction, which under Gumbel distributed bids are calculated as:

$$r_{vi}^{t}=\frac{1}{\mu}\ln\left(\sum_{g}H_{g}^{t}\exp\mu\left(b_{h}^{t}+B_{gvi}^{t}\right)\right)+\frac{\gamma}{\mu}\qquad\forall v,i,t \tag{6}$$

where $\gamma=0{,}577$ is the Euler's constant. Here, we observe that bids and rents are functionally dependent by Equation 6, therefore rents are hedonic price functions directly correlated with real estate attributes and access measures. It is worth commenting that that this correlation makes very difficult to calibrate location choice models based on maximizing the utility function of Equation 3, which yields the classical choice logit probability model, because attributes and rents are mutually correlated. In contrast, we note that this problem disappears in the estimation of bids using the bid-auction probabilities (Equation 5), because bids do not depend on rents. Additionally, the number of alternatives in Equation 5 is N_h, a small number compared with the zone-real estate combinations ($N_i \cdot N_v$) that define the options' set in the location choice model. Finally, the bid-auction and the choice probabilities are consistent only under equilibrium conditions; under dis-equilibrium the auction process holds. These are arguments that make the bid auction model easier to estimate and theoretically supported.

Real estate supply model

In this section we model the change in real estate supply between two time periods, based on the assumption that suppliers maximize a random profit with perfect knowledge of the total demand at each period but unable to perfectly predict demand and prices at each sub-market *(v,i)* in the next period. The model is:

$$V_{vi}^{t}=V_{vi}^{t-1}+F\cdot\left(H^{t}-H^{t-1}\right)P_{vi}^{t}\qquad\forall v,i,t \tag{7}$$

where P_{vi}^{t} is the probability that suppliers will develop new real estate unit type (*v,i*) in period *t* , which will be available for auction in period *t+1*. Since total demand variation between these periods ($H^{t}-H^{t-1}$) is exogenous, a factor *F=1* assumes perfect knowledge of total demand and that is matched by the supplier (called the perfect foresight assumption) and any difference between supply and demand at time period is exactly replicated along all future periods. Alternatives assumptions may be assumed modeling the *F* factor (see for example the myopic assumption in Martínez and Hurtubia, 2006). Hereafter in this paper we assume *F=1*.

The profit obtained from supplying a real estate unit (*v,i*) in period *t,* denoted π_{vi}^{t}, is assumed a random variable distributed independent and identical Gumbel with parameter λ. Then, the probability that a unit will be developed in a specific option *(v,i)* is given by:

$$P_{vi}^{t}=\frac{\rho_{vi}^{t}\exp\left(\lambda\pi_{vi}^{t}\right)}{\sum_{v'i'}\rho_{v'i'}^{t}\exp\left(\lambda\pi_{v'i'}^{t}\right)}\qquad\forall v,i,t \tag{8}$$

Here we introduce the cutoff factors, denoted by ρ_{vi}^{t}, of the constrained multinomial logit model (CMNL) proposed by Martínez et al. (2009), which provides a mean to represent the feasible domain for developers of real estate units, accounting for example for zoning regulations that may prevent some real estate types to be developed in specific zones. This feature of the model is very useful to represent in the model the highly diverse and numerous regulations that apply to urban development in most modern cities.

Profit functions are modeled as follows:

$$\pi_{vi}^{t} = \pi(r_{vi}^{t-1}) \qquad \forall v, i, t \tag{9}$$

where profits depend on lagged rents. In the application shown below we assume linear profit functions.

3. QUASI-EQUILIBRIUM AUCTION ALLOCATION MODEL

Following the bid-auction theory of Alonso (1964), agents are assigned to available locations according to the best bidder rule, which under equilibrium conditions the resulting allocation complies with the maximum utility rule followed by the random utility theory. Hence the two approaches collapse into one unified model. The Random Bidding and Supply Model (RB&SM) (Martínez and Henríquez, 2007) uses this unified theory for the logit model to solve the equilibrium problem's unique solution, which is implemented in the operational software MUSSA (renamed CUBE LAND, see Martínez and Donoso, 2010).

The allocation of households and firms by the best bidder rule requires that, at any point in time, willingness to pay values adjust to the prevailing supply conditions such that every one is allocated at equilibrium. This adjustment induces further adjustments on rents, following Equation 6, because rents depend on bids by the logsum function. Such adjustments for the equilibrium were studied in the RB&SM, where equilibrium implies the following conditions: (a) total supply equals total demand; (b) all externalities are solved to an state where no one is better off by a unilateral relocation; (c) reservation utilities adjust to comply with the clearing condition that all agents are allocated; the approach assumes that transport accessibility indices are exogenous.

In this section we present a model where these conditions are relaxed, yielding a model that predicts a system dynamics following a path of quasi-equilibrium states that are consistent with the bid-choice theory. The resulting model retains the property uniqueness of the solution, which is only dependent on the initial conditions and the length of the time period. This feature provides stability to the model predictions, in contrast with other simulation models with are path dependent, i.e. each run predicts different solutions on development, location of agents, densities, agglomeration economies and rents.

The total supply-demand equilibrium condition (a) at any point in time is fulfilled by Equation (7). We consider that this assumption of *suppliers' perfect foresight* of total demand because, for a short period interval, aggregated demography is highly predictable. Then, the expected number of households per cluster allocated at each real estate option (*v,i*) is given by: $H_{hvi}^{t}=V_{vi}^{t-1}p_{h/vi}^{t}$. Since bid-auction probabilities add up to one across consumer's clusters, it follows that total number of households allocated to a given supply option complies with the supply V^{t-1} available at every period *t* (produced with one period delay).

The relaxation of the externalities adjustments (b) is performed by the usual approach of delaying zone attributes by one time period in the willingness to pay function, as shown in Equations 3 and 4; the same approach is used for updating accessibility. This can be supported by the argument that these variables are aggregated states of the system that take some time to be informed to the consumers. Nevertheless, this is also debatable because to some extend changes are anticipated by rational individuals.

In this section we propose a new methodology to adjust bids relaxing condition (c) at every point in time, i.e., how to adjust $b_h^t = y_h - U_h^t/\beta_{1h}$ in Equation 5 for each household cluster, and for each time period. With exogenous incomes levels (y_h), this represents the adjustment in reservation utility levels by each cluster.

Consider the RB&SM equilibrium condition that assures that every household is allocated: $\sum_{vi} H_{hvi}^{t}=\sum_{vi} V_{vi}^{t}p_{h/vi}^{t}=H_h^t$. To fulfill this condition reservation utilities of all agents are simultaneously adjusted in the RB&SM by solving a complex fixed-point problem; this condition assumes perfect agents' information of the market, which is plausible in the long term. Our approach defines a quasi-equilibrium assuming imperfect information due to the short term between two consecutive periods: each agent adjusts s/her reservation utility at the maximum level that assures her/him is allocated somewhere at each point in time while she/he ignores others' utilities. This is yield by the following condition:

$$\sum_{vi} H_{hvi}^{t}=\sum_{vi} V_{vi}^{t-1}\, q_{h/vi}^{t}=H_h^t \qquad (10)$$

where the quasi-equilibrium bid-auction probability *q* is defined as:

$$q_{h/vi}^{t}\equiv\frac{H_h^t \exp\mu(b_h^t+B_{hvi}^t)}{H_h^t \exp\mu(b_h^t+B_{hvi}^t)+\sum_{g\neq h} H_g^t \exp\mu(b_g^{t-1}+B_{gvi}^t)} \qquad \forall h,v,i,t \qquad (11)$$

Replacing Equation 11 in 10 and solving for b_h^t yields the individuals' reservation utility b_h^{*t}, which represents her/his ex-ante evaluation of the maximum attainable utility based on past information of all other agents' behavior. Analytically, Equation 11 cannot be solved for b_h^t, we can only define the following non-linear fixed-point problem:

$$b_h^{*t}=-\frac{1}{\mu}ln\sum_{vi}\frac{V_{vi}^{t-1}\exp(\mu B_{hvi}^t)}{H_h^t \exp\mu(b_h^{*t}+B_{hvi}^t)+\sum_{g} H_g^t \exp\mu(b_g^{t-1}+B_{gvi}^t)} \qquad \forall h,t \qquad (12)$$

It is worth noting that under these individuals' reservation utilities the RB&SM equilibrium condition does not hold, i.e. $\sum_{vi} V_{vi}^{t}\cdot q_{h/vi}^{t}\,(b^{*t})\neq H_h^t$. This inequality reflects the situation that an erroneous estimation of reservation utilities by imperfectly informed agents yields potentially excessive high or low of demand for each location option.

A simple approximation of the solution is obtained by taking b_h^{t-1} as a proxy of b_h^t in the denominator (only) of Equation 12, which yields the following linear estimate for b_h^{*t}:

$$b_h^{*t} = -\frac{1}{\mu} ln\left[\sum_{vi} V_{vi}^{t-1} exp\left(\mu B_{hvi}^t - ln\left(\sum_g H_g^t \exp\left(\mu b_g^{t-1} + \mu B_{gvi}^t\right)\right)\right)\right] \quad \forall h,t \qquad (13)$$

The auctioneer observes these bids and allocates real estate to the maximum bidder, yielding locations rents estimated by the following expected maximum bid:

$$r_{vi}^t = \frac{1}{\mu}\ln\left(\sum_g H_g^t \exp\mu(b_h^{*t} + B_{gvi}^t\right) + \frac{\gamma}{\mu} \qquad \forall v,i,t \qquad (14)$$

with γ the Eulers' constant.

The quasi-equilibrium land use model algorithm

Hereafter we linear in parameters bid functions and $\mu = 1$, because bid parameters are assumed scaled by μ in the estimation procedure.

The model is implemented by the following calculation procedure:

Read variables at t=0 : $(r_{vi}^0)_{v,i}$, $(z_i^0)_i$, $(acc_{hi}^0)_{h,i}$, $(V_{vi}^0)_{v,i}$

For t=1,2,...,T sequentially:

1. Read new scenario t :

   $(H_h^t)_h$ and regulations to calculate supply cutoff

2. Calculate bids (Equations 4 and 13):

$$B_{hvi}^t = B\left(\left(z_{i'}^{t-1}\right)_{i'}, \left(acc_{h'i'}^{t-1}\right)_{h'i'}\right) \quad \forall h,v,i$$

$$b_h^t = -ln\left[\sum_{vi} V_{vi}^{t-1} exp\left(B_{hvi}^t - ln\left(\sum_g H_g^t exp\left(b_g^{t-1} + B_{gvi}^t\right)\right)\right)\right] \quad \forall h$$

3. Allocate residents and firms replacing *p* by *q* in Equations 5:

$$H^t_{hvi} = V^{t-1}_{vi} q^t_{h/vi}\left(\left(H^t_{h'}\right)_{h'}, \left(b^t_{h'}\right)_{h'}, \left(B^t_{h'vi}\right)_{h'}\right) \quad \forall h, v, i$$

4. Calculate supply (Equations 7, 8 and 9)

$$^{\text{º}}V^t_{vi} = V^{t-1}_{vi} + (H^t - H^{t-1}) P^t_{vi}\left(\left(r^{t-1}_{v'i'}\right)_{v'i'}, \left(z^{t-1}_{i'}\right)_{i'}\right) \quad \forall v, i$$

5. Update attributes and rents (Equations 14):

$$z^t_i = z^t_i\left(\left(H^t_{h'i}\right)_{h'}\right) \quad \forall i$$

$$r^t_{vi} = r^t_{vi}\left(\left(H^t_{h'}\right)_{h'}, \left(b^t_{h'}\right)_{h'}, \left(B^t_{h'vi}\right)_{h'}\right) \quad \forall v, i$$

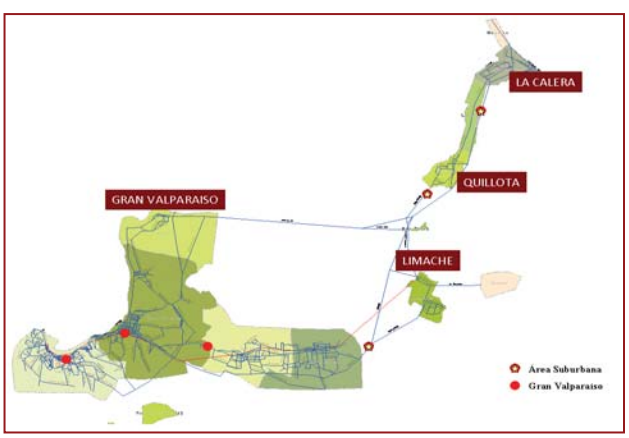
6. Run the transport model to update accessibility indices.

It can be seen that the algorithm can be easily set differently by switching steps 4 and 5, but this implies different assumptions about suppliers' behavior: in this case suppliers are informed instantaneously of rents and land use, then supply probabilities depends on rents and location attributes observed without lags. We leave this option for empirical testing but it is not considered hereafter.

4. APPLICATION

We applied this model to study the extension of the commuting service of the MERVAL Train Company from Valparaíso – Limache corridor up to La Calera as depicted in Figure 1. This new service would operate from the Valparaiso urbanized area (called Gran Valparaíso or "the city") to La Calera, along the suburban area with different towns (called "Area Suburbana"), currently served by buses from the terminal located in Limache town. In this application real estate types were not considered due to the lack of data; hence households and firms were allocated to zones.

Figure 1: Valparaíso – La Calera corridor



4.1. Parameters estimation

The model was applied to Valparaiso – La Calera corridor and surrounding areas where daily commuting trips takes place by car, bus and train. The location attributes considered in the land use model are: built area (*SC*) used by industry (*IN*), retail (*CO*), services (*SE*), education (*ED*), health (*SA*) and other (*OT*); total zonal space (A_i); households total by zone (H_{hi}) and of high-income (H_{HIGHi}) and low-income (H_{LOWi}); dummies ($1_{i\in \text{area}}$); the residential floor space rent per square meter (r_i); and morning peak hour travel time by income level of commuter's household and origin zone (tv_{hi}) which is averaged along destination zones. Table 1 shows the maximum likelihood estimates of willingness to pay functions using residential and non-residential bid-auction probabilities (Equation 5). The goodness of fit of the model is described by the R^2 coefficient of the regression between the observed and estimated number of households in *t=0,* i.e. we regressed $H_{hi}^0 = \theta_0 + \theta_1 \widehat{H}_{hi}^0$; this coefficient is presented for the city zones (R^2(City)) and for the towns (R^2(Towns)) separately.

The parameter estimates verify the expected influence in location choice with right signs. The R^2 goodness-of-fit coefficients of the residential model are high, near 90%, except in the case of high-income households located in towns (68%). This indicator is also high for Retail, Services and Other activities, although it is lower in the remaining activities. Constants of high-income groups are set equal to zero, thus the negative sign of constants reflect relatively lower willingness to pay of low and middle-income groups compared with high-income groups (*ceteris paribus* other attributes). For low-income residents the most attractive attributes are commuting time and location of industry in the same area, while for middle income the retail sector is the most attractive one. Commuting travel time is highly significant in all income groups and negative, as expected, influencing more to those towns located farther away from the city and to low and medium income groups than in the high one. Socio-spatial effects or location externalities among residents are present in middle and high-income groups: the middle-income groups are willing to pay an extra amount in zones with high-income families, while for high-income groups the percentage of low-income households in the same zone reduce their bids. Most of non-residential activities are highly and positively influenced by the presence of households in the zone.

Table 1: Willingness to pay parameters by agent type [(a)]

Attribute	Residential market			Non-Residential market					
	Low-Income	Medium-income	High-income	Industry	Retail	Services	Education	Health	Other
Constant	-2,235	-3,438				-5,849			
H_{HIGHi}/H_i		1,436							
H_{LOWi}/H_i			-7,891						
$(H_{LOWi}/\Sigma_h H_{hi})*1_{i\in City}$				1,296					

Attribute	Residential market			Non-Residential market					
	Low-Income	Medium-income	High-income	Industry	Retail	Services	Education	Health	Other
$(H_{LOWi}/\Sigma_h H_{hi})*1_{i\in Towns}$				2,536					
$H_{HIGHi}/\Sigma_h H_{hi}$					2,087				1,736
H_i/A_i							246,714		
$(H_i/A_i)*1_{i\in City}$								251,287	
$(H_i/A_i)*1_{i\in Towns}$								9,176	
$ln(SC_{INi})$	0,050	-0,039							
$ln(SC_{COi})$		0,219				0,516			
$ln(SC_{SAi})$	0,046								
SC_{COi}/A_i			8,103						
SC_{SAi}/A_i			1,364						
$SC_{EDi}/\Sigma_{i'} SC_{EDi'}$			1,231						
$tv_{hi}*1_{i\in Towns}$	-0.045	-0,050	-0,023						
$1_{i=Valparaíso\ Port}$				-1,128			-0,672	-1,161	
$1_{i=Reñaca}$			4,906						
$1_{i=La\ Calera\ downtown}$						-1,408			
$R^2(U)$	0,86	0,89	0,92	0,31	0,91	0,98	0,57	0,44	0,95
$R^2(S)$	0,87	0,90	0,68	0,83	0,87	0,93	0,64	0,02	0,89

(a) All parameters significant at the 95% confidence level.
(b) $1_{i\in City}$, $1_{i\in Towns}$, $1_{i=Valparaiso\ Port}$, $1_{i=Reñaca}$ and $1_{i=La\ Calera\ downtown}$ are zonal dummies.
(c) Valparaíso Port is the downtown of the city and Reñaca is a beach residential zone.
(d) The thresholds that define income levels are: \$245.463 and \$866.340, which are monthly incomes in Chilean pesos of June 2010.

Rents can be estimated using Equation 6 and bids parameters of Table 1. Only residential rent model was estimated due to lack of non-residential rent data. We specified linear bid components that are constant across households in the following functional form:

$$r_i = \frac{1}{\mu} ln\left(\sum_g H_g \exp(B_{hi})\right) + \alpha_o + \sum_k \alpha_k z_{ki} \quad (15)$$

The ordinary least squared estimates of parameters are shown in Table 2. Since the logsum coefficient is highly significant and positive, we conclude that rents follow the maximum bid rule as expected theoretically. Moreover, commuting time is also significant and with right negative sign in its parameter. Then, it is worth noting that in this model the accessibility measure influences rents in two complementary ways: through the logsum term, where the effects are differentiated by household type's willingness to pay, and by an additional term in the rent function that is common to all agents.

Table 2: Residential real estate rent model

Attribute	**Coefficient**	**t-test**
Constant	0,1273	15,65
$ln\left(\sum_g H_g \exp\left(\mu B_{gi}\right)\right)$	0,0150	2,25
$tv_i * 1_{i \in Towns}$	-0,0012	-2,33
R^2	0,47	

Aggregate real estate supply logit models with linear profit functions were estimated independently for residential and non-residential submarkets, using the maximum likelihood method. Due to the lack of supply information by real estate types, supply models in Equation 7 were simplified to $V_i^t = H^t P_i^t$ where P_i^t is the aggregate probability of developing location units in zone i. The estimated parameters of both models are shown in table 3.

Table 3: Real estate supply models

	Residential market		Non-residential market	
Attribute	**Coefficient**	**t-test**	**Coefficient**	**t-test**
r_i	3,373	97,2	2,472 (b)	27,5
H_i/A_i			419,48	9,04
$\Sigma_i SC_{fi}/A_i$ (a)	-0,258	-5,9		
SC_{EDi}/H_i	1,013	944,6		
$1_{i \in City}$	-0,902	59,6		
$1_{i=Valpo\ center}$			7,43	11,92
Nr observations	32		32	
Log-likelihood average	-32129		-74,25	
R^2	0,12		0,87	

(a) This sum includes industry, retail, services, education, health and other

(b) Residential rent is used as a proxy variable for non-residential one

While the R^2 coefficient of the non-residential model is very high, it is low in the other model. This is explained by the very high concentration of non-residential development in the Valparaíso city center, captured by a specific dummy variable. A significant and positive rent parameter was obtained in both models, which supports the hypothesis that suppliers maximize profit in both markets. Other agglomeration attributes were highly significant showing that residential and non-residential supply markets influence each other.

Forecasting

The land use model with the estimated parameters shown in tables 1 to 3, and the quasi-equilibrium land use model algorithm were used to forecast the impacts of extending the rail mode currently operating between Gran Valparaíso-Limache up to La Calera (Figure 1) in period 2010-2018.

The land use model allocates households and non-residential built area to consumers based upon exogenous estimation of the total number of households by income level and total built area by activity type. The first was calculated as follows: the total number of households provided by the National Statistical Institute (INE) as official demographic projection was split into income groups by applying an income distribution model. For non-residential projections of total activity, which are measured by total built areas, we applied an econometric model based on the assumption of 4.5% increase in the Regional Internal Gross Product (PIB). Table 4 and 5 show both estimates:

Table 4: Number of household projections per income category

Year	Low income	Medium income	High income	Total
2010	98.200	200.533	37.815	336.549
	29%	60%	11%	100%
2018	92.368	229.408	54.613	376.389
	25%	61%	15%	100%
Annual Grow Rate in period 2010-2018	-0,8%	1,7%	4,7%	1,4%

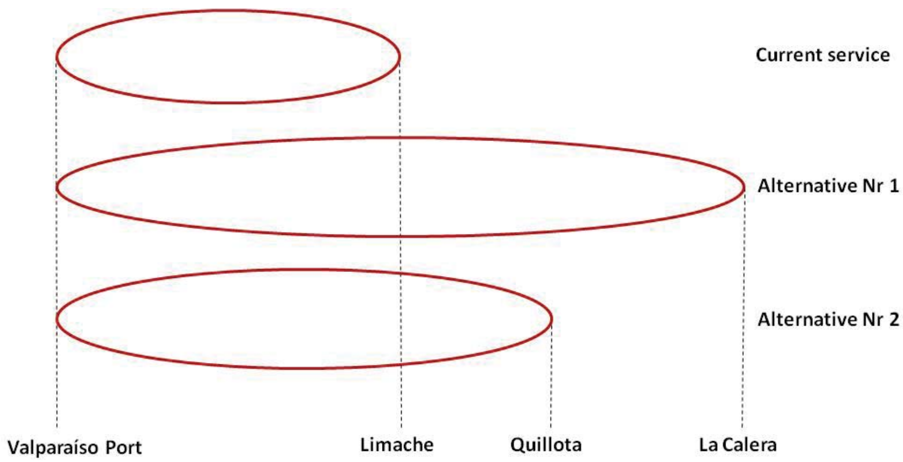
Table 5: Non-residential land use projection

Year	Industry	Retail	Services	Education	Health	Other	Total
2010	1.069.549	2.788.326	1.228.693	1.355.156	279.448	1.700.803	8.421.975
	13%	33%	15%	16%	3%	20%	100%
2018	1.176.555	3.172.665	1.391.696	1.422.020	289.020	1.938.672	9.390.628
	13%	34%	15%	15%	3%	21%	100%
Annual Grow Rate in period 2010-2018	1,2%	1,6%	1,6%	0,6%	0,4%	1,6%	1,4%

These projections show that total households of medium and high income levels increase in absolute and relative terms during the period 2010-2018, with a contrary tendency in the low income group. In the non-residential market, all activities will expand particularly Retail, Services and Other.

The quasi-equilibrium land use model interacted with a four-stage transport model previously implemented; the first provided location estimates to the second, which in turn returned travel time predictions. Both models are applied jointly to evaluate time benefits of two alternative train services, a short extension up to Quillota town (Alternative Nr 1) and a long one up to La Calera town (Nr 2). Both alternative services are compared to the current service up to Limache town, as shown in Figure 2. In all alternatives the train headway is 12 minutes in the morning peak period and 24 minutes in the off-peak. Fares remain the same for all alternatives an equal to the current bus-train combination service.

Figure 2: Alternative services studied



Simulations of the land use – transport system were performed for each alternative and year along the study horizon 2010-2018, distinguishing morning peak and off-peak hours in transport simulations. For each year, total households by income category and total non-residential land footage define common exogenous conditions for all simulations. Additionally, all projections are based upon the same initial land use distribution at 2010, which is a benchmark for comparison purposes (Figures 3 and 4).

Figure 3: Nr of households by comuna in 2010

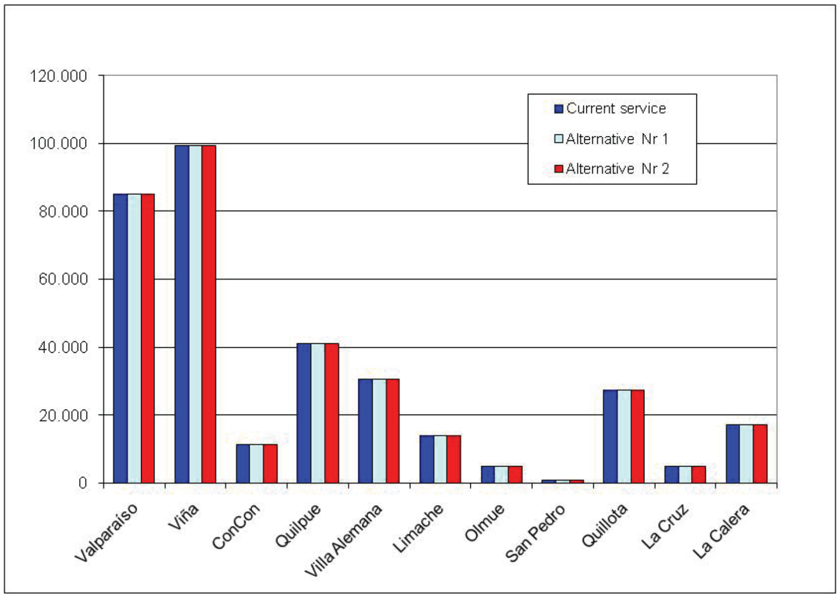
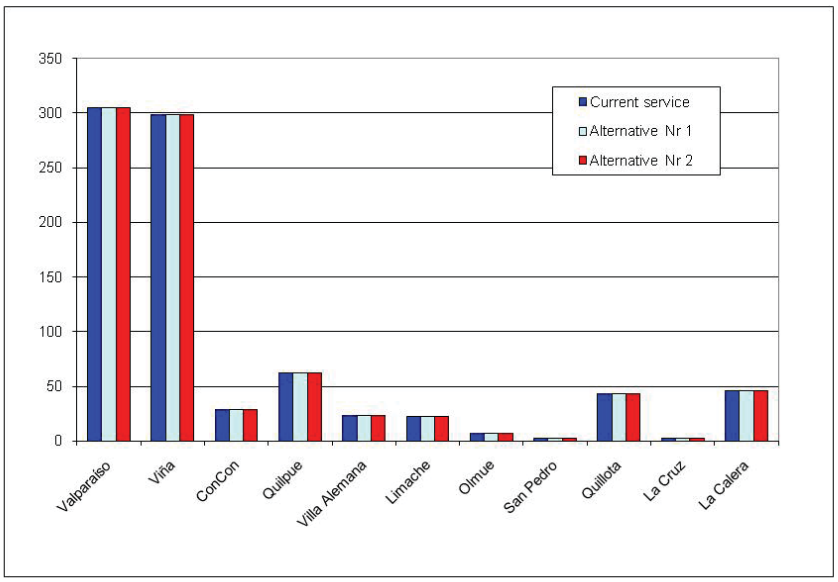


Figure 4: Non-residential built area in Ha by comuna in 2010



The results of the simulations for 2018 are shown aggregated by area in Tables 6 and 7 and by comuna in Figures 5 and 6, for each alternative service.

Table 6: Forecast of number of households in 2018 by income, area and alternative rail service

Service	Area	Low income	Medium income	High income	Total
Current service	City	73.076	181.579	35.367	290.022
	Towns	19.130	47.666	19.571	86.367
	City + Towns	92.206	229.245	54.938	376.389
Alternative Nr 1	City	67.994	178.711	39.840	286.545
	Towns	24.230	50.576	15.037	89.844
	City + Towns	92.225	229.287	54.877	376.389
Alternative Nr 2	City	68.729	179.033	39.438	287.200
	Towns	23.497	50.249	15.442	89.189
	City + Towns	92.227	229.282	54.880	376.389

Table 7: Forecast of non-residential built area in Ha in 2018 by activity, area and alternative rail service

Service	Area	Industry	Retail	Services	Education	Health	Other	Total
Current service	City	90	250	129	124	25	156	774
	Towns	26	68	11	18	4	38	165
	City + Towns	116	318	139	142	29	194	939
Alternative Nr 1	City	81	251	127	121	25	156	761
	Towns	36	67	12	21	4	38	178
	City + Towns	117	318	139	142	29	194	939
Alternative Nr 2	City	84	252	127	122	25	156	765
	Towns	33	67	12	20	4	38	174
	City + Towns	117	318	139	142	29	194	939

Figure 5: Forecast of number of households by comuna in 2018

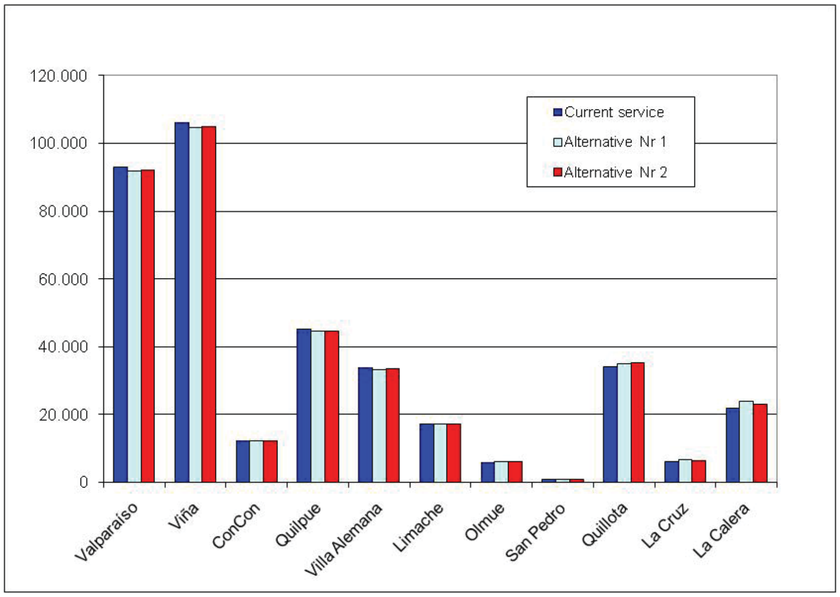
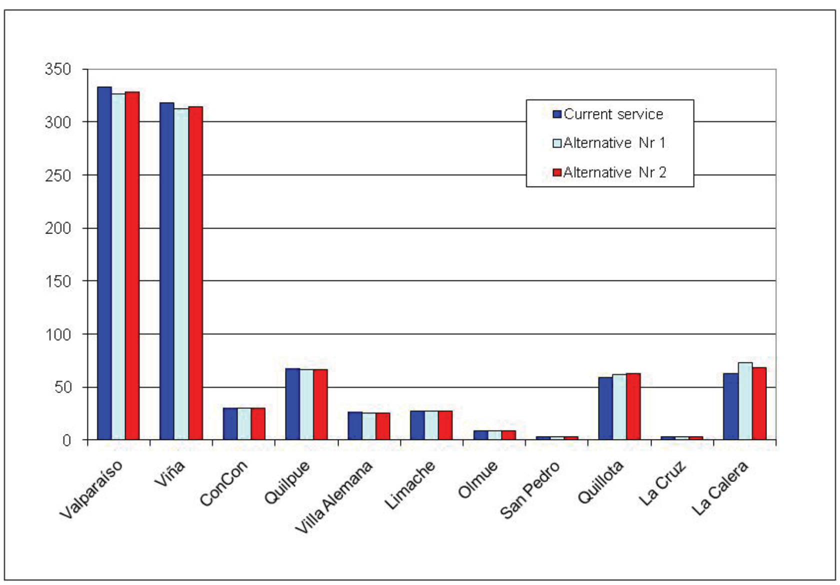


Figure 6: Non-residential built area estimates in Ha by comuna in 2018



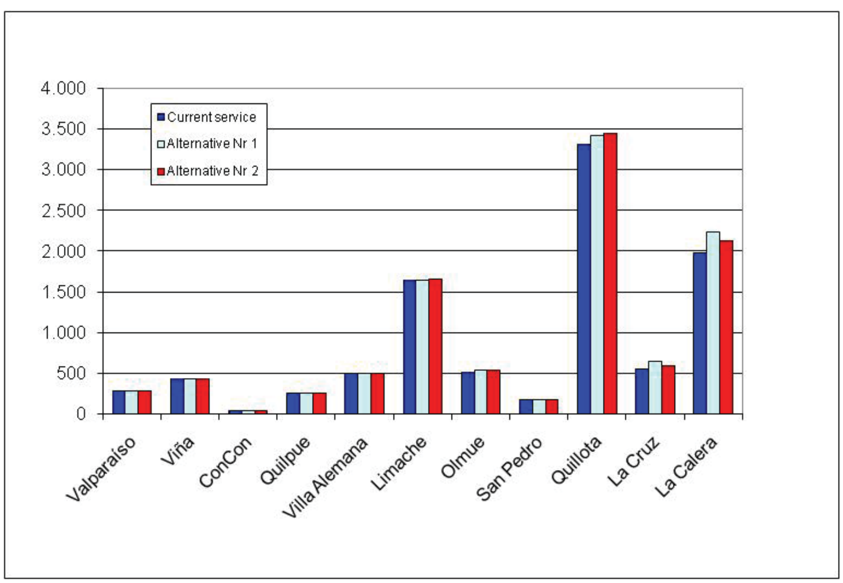
A first check is that the model complies with exogenous projections of total agents in "City+Towns" area. Demand increases in the towns, more so in the longest service up to La Calera (Alternative Nr 1) and in the low and middle-income groups, which is consistent with the fact that the better public transport train service provided is more attractive for them whom bid higher for this land with better rail service. Conversely, the high-income group concentrates in the urban area. Forecasts of non-residential land use show a migration towards towns compared with the city, which is pronounced for longest train service.

Table 8 and Figure 7 present the estimates of commuting trips forecasts for each alternative service in 2018. It can be observed an increase in travel commuters in new services compared with the current one, which is higher for the further towns of Quillota and La Calera, particularly so for the long direct service to these towns. The urban area travel demand remains constant.

Table 8: Commuting Trips by comuna and period in 2018

Comuna	Morning peak hour			Off peak hour		
	Current service	Alt. Nr 1	Alt. Nr 2	Current service	Alt. Nr 1	Alt. Nr 2
Valparaíso	286	286	286	224	224	224
Viña	427	427	427	242	242	242
Concón	41	41	41	33	33	33
Quilpué	250	250	250	167	167	167
Villa Alemana	493	493	493	116	116	116
Limache	1.640	1.647	1.656	1.470	1.473	1.479
Olmué	509	542	543	473	479	480
San Pedro	172	177	177	157	160	159
Quillota	3.304	3.423	3.443	2.876	2.884	2.935
La Cruz	548	645	587	546	558	535
La Calera	1.975	2.240	2.124	1.754	1.887	1.775
Total	9.646	10.171	10.027	8.058	8.223	8.145

Figure 7: Commuting trips by comuna and morning peak period in 2018



5. CONCLUDING REMARKS

The concept of equilibrium becomes operational by the set of conditions that defines this market state, but such conditions may vary from a static equilibrium, where all endogenous variables attain a steady state conditional on exogenous inputs, to a more dynamic definition allowing for delays in market adjustments that we call quasi-equilibrium. The former is plausible in the long-term while the latter is consistent with a shorter-term analysis. Nevertheless, as long as prices and the allocation of agents are modeled as the result from some set of market conditions it is a model of equilibrium; on the contrary, if these outputs are the result of independent sequential processes we consider them a simulation model.

This paper proposes and applies a quasi-equilibrium model for an open urban market, originally designed for a city and towns region with high commuting, but applicable for a wide range of urban and sub-urban contexts. The model produces endogenous estimates of land rents, real estate development and residential (and non-residential) allocation at each point in time, which allow for the estimate of excess demand and supply consistent with random bid-auction theory. The model is stable and path independent, having a unique solution that depends on initial conditions and exogenous estimates of total number of households and non-residential demand. Calculation time is negligible and implementation is simple.

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