Dynamic Multiline Vehicle Dispatching Strategy in Transit Operations

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Abstract

Providing regularity in buses' operation in high-frequency services is essential to offer a good quality of service to users. If buses are not dispatched at regular headways from the terminal, headway irregularity will gradually increase along the line. In this work, we study a vehicle dispatching problem in which multiple lines start their operations from a common terminal where buses can interchange between lines. The model simultaneously decides the ideal dispatching headway for each line and assigns the following arriving buses to the terminal its line to operate and its corresponding dispatching time. The objective is to minimize the dispatching interval's deviation from an ideal headway that is dynamically updated based on the system's status. We formulate our problem as a Mixed-integer quadratic problem and adopt a rolling horizon policy to cope with the dynamic and stochastic environment of public transit systems. We prove that a bus assignment that satisfies the FIFO discipline is an optimal solution for the proposed problem. We evaluate our model in a simulation environment under different operational conditions and study the incremental benefits of allowing different flexibility schemes. Our results show that a full flexibility scheme where buses can freely interchange between lines reduces the coefficient of variation of dispatch headways and improves frequency compliance by nearly 20% when compared with the case where buses are restricted to operate in a single line. It also outperforms a myopic heuristic that adopts a *a priori* target headway.

Keywords: Multiline transit operations; headway regularity; Bus bunching, Bus dispatching.

1 Introduction

High-frequency transit systems operating at regular intervals are essential to offer a good quality of service to users. Running at even headways reduces waiting times for passengers. It also increases comfort by allowing more homogeneous loading between buses by preventing the classic phenomenon observed when bus bunching occurs where most passengers wait for long periods to board full buses. Moreover, regular headways positively affect travel, and cycle time, making driver shift planning and operation more reliable (Muñoz et al, 2020).

One way to improve regularity on a high-frequency line is by applying different control strategies along the route. Ibarra-Rojas et al (2015) presents a complete literature review where real-time control strategies such as holding, stop-skipping, bus speed regulation and Traffic Signal Priority are discussed. Most of these studies have focused on controlling a single line in isolation.

If buses are not dispatched at regular intervals from the terminal, headway irregularity will increase along the line (Barnett, 1974; Daganzo, 2009; Berrebi et al, 2015). Figure 1 extracted from Delgado et al (2016) shows this phenomenon, for a specific line in the Transantiago system in Santiago. Two aspects stand out in the figure. First, buses are rarely dispatched regularly from the terminal. Second, how the coefficient of variation of headways (COV) amplifies along the route if regularity is not achieved at the terminal.

Dispatching buses from terminals is not an easy task. Transit operators should care about fulfilling the operational plan's frequency while making dispatches as regular as possible. If not, transit agencies, such as in Santiago de Chile, establish a series of fines if the operators do not comply with minimum standards of frequency and regularity (Muñoz et al, 2014). In practice, sometimes dispatching at regular headways is impossible due to buses' absence caused by their operation delay. An additional aspect that makes bus dispatching a complex task is that the transit operator must dynamically make dispatch decisions for multiple lines that start their operation from the same terminal during a certain operational period. Each of these lines must meet its own operating plans determined by the frequency of buses to be offered in each period. In this way, dispatching buses from a terminal consists of determining to which line assign and when to dispatch the next arriving bus to the terminal.

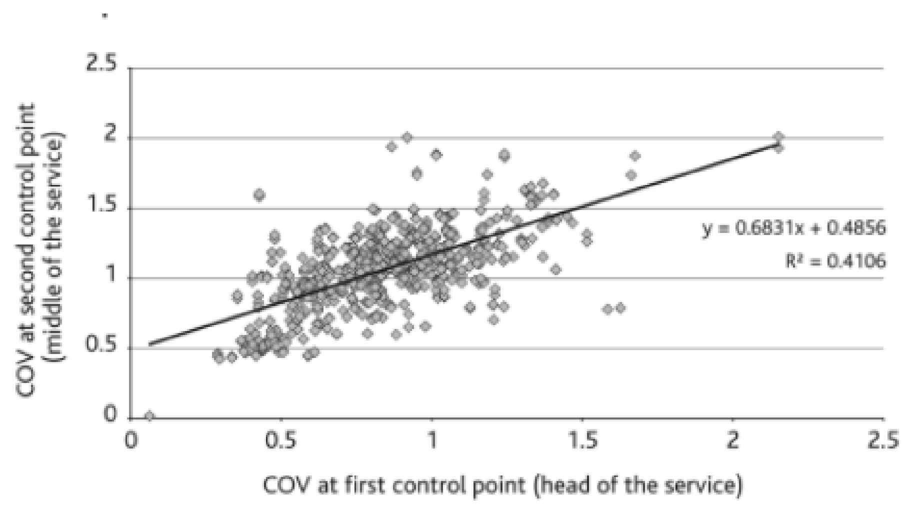


Figure 1: Correlation between COV of headways at the terminal and middle of the route. Source (Delgado et al, 2016)

In this work, we introduce a real-time Multiline Vehicle Dispatching Problem (MLVDP), which dynamically assigns each bus that is about to arrive at the terminal, its line to operate in the next trip and its dispatch time. The objective seeks to comply with the frequency established in each line's operation program while minimizing the dispatching interval's deviation from the ideal one. Thus, we abandon the idea of *a priori* target headway (Daganzo, 2009; Xuan et al, 2011; Gkiotsalitis and Van Berkum, 2020) and consider this ideal headway as a decision variable of our optimization model. This headway depends on the remaining number of buses to be dispatched and the time interval before the operational period ends.

The remainder of this article is organized as follows. In Section 2 we present the related literature review. In Section 3 we formally state the Multiline Vehicle Dispatching Problem (MLVDP), derive a FIFO dispatching discipline and propose a Rolling Horizon dispatching policy. Section 4 presents our simulation experiments and results. Finally, Section 5 conclude our work and suggest future research.

2 Literature Review

Real-time control strategies are among the most effective measures to fight bus bunching and its adverse effects on transport users in headway-based services. The literature has been prolific in studying and proposing different corrective strategies, which can be classified into two groups: station control and inter-station control. In the first group, we find holding (Eberlein et al, 2001; Sun and Hickman, 2008; Bartholdi III and Eisenstein, 2012; Berrebi et al, 2015; Sánchez-Martínez et al, 2016; Dai et al, 2019), stop-skipping (Sun and Hickman, 2005; Cortés et al, 2010; Zhang et al, 2020) and boarding limits (Delgado et al, 2012; Bueno-Cadena and Muñoz, 2017). As for inter-station control strategies, speed control (Daganzo and Pilachowski, 2011; Bian et al, 2020) and Traffic Signal Priority strategies (Delgado et al, 2015; Chow et al, 2017) stand out in the literature. Despite the important contributions of these studies, most of them have focused on controlling a single line at a time.

The real-time control strategies considering multiple lines that interact have been studied more recently in the literature. In most studies, the interaction between lines arises because dwell times at a stop are affected by waiting passengers who are indifferent between multiple lines and by transfers. Hernández et al (2015) proposed an optimization model based on holding strategy for a corridor with multiple bus lines. The authors apply their model employing a rolling horizon policy and compare the impacts of centralized versus decentralized control schemes. Seman et al (2019) presented a non-linear programming formulation for the same problem. Their numerical results show the superiority of the proposed multiline headway control over its single-line counterpart. Argote-Cabanero et al (2015) introduced a simple dynamic control that shows resilience under disruptions while the complexity of the problem remains stable as the system size grows. Schmöcker et al (2016) studied the effect of common lines on bus bunching dynamics. In their formulation, the authors consider the possibility of bus overtaking and include a passengers' route choice model based on hyperpaths. The authors demonstrate that common lines will have a positive effect on service regularity when overtaking is allowed.

Petit et al (2019) proposed a multiline bus substitution strategy, where standby buses are shared across lines. Under this setting, the transit agency has to decide how to dispatch (insert) standby buses and where to reposition the retired ones. The authors used continuum approximation to model the problem and proposed an Approximate Dynamic Programming algorithm to solve the infinite-horizon stochastic dynamic program.

To our knowledge, Gkiotsalitis and Van Berkum (2020) is the only work that explicitly addresses the problem of bus dispatching from a terminal. The authors considered a single line and proposed a rolling-horizon optimization model that determines buses' dispatching time to deviate from *a priori* target headway as little as possible. To efficiently solve the problem, the authors developed a convex reformulation of the problem.

Our work extends Gkiotsalitis and Van Berkum (2020) paper by considering the problem of bus dispatching from a terminal where multiple lines begin their service. In this case, the interaction between lines is manifested in sharing or exchanging buses between lines. In particular, we consider the following to be our main contributions:

- We formulate, study and solve a model to dispatch buses from a terminal where multiple lines start their operations. To our knowledge, ours is the first model to integrate bus assigning and dispatching times decisions while abandon the idea of *a priori* target headway in a setting where multiple lines can interchange their buses.
- We include a complete mathematical analysis and prove that a FIFO discipline is an optimal dispatching policy.
- We empirically study the value of i) having more accurate travel time predictions and ii) allowing different flexibility schemes. In our case, we define flexibility as the possibility to interchange or share buses to allow them to operate in different lines.

3 Problem Statement

Now, we formally state the Multiline Vehicle Dispatching Problem (MLVDP), which dynamically assigns the line to operate and the dispatching time for the following arriving buses to the terminal.The objective of the MLVDP is to minimize the dispatching interval's deviation from the ideal headway, which is dynamically updated based on the current status of the system.

3.1 Problem definition

Consider a terminal where a set of L lines start their operations. Every line is defined by the set of buses in operation, denoted by $\mathcal{B}_l$ and its operational plan during each period (e.g., morning peak), represented by its nominal frequency fr_l. At every point in time t, we know the time interval H before the period ends. Also, for every line $l \in L$, we have information of the time elapsed $d_{l,0}$ since the last bus (denoted by 0) was dispatched, and the number of buses r_l that need to be dispatched during the next H units of time to fulfill the operational plan. Additionally, we consider a subset of buses $B_l \subseteq \mathcal{B}_l$ that represents the buses contemplated in the planning horizon. For each bus $b \in B_l$ we estimate in real-time, based on GPS data and historical information, the remaining time $f_{l,b}$ for the arrival of each of these buses to the terminal.

Based on the above information, every time a bus $k_s \in \mathcal{B}_s$ arrives to the terminal the decision maker needs to make the following decisions:

- First, decide the optimal dispatching headway for each line h_l^* considering that r_l buses needs to be dispatched before the end of the period.
- Second, simultaneously assign the next arriving bus (k_s) as well as all buses included in $B_l, \forall l \in L$ to a line $l \in L$ to operate during the next trip. Let $x_{(s,k_s)(l,k_l)} \in \{0,1\}$ be a binary variable, which is equal to 1 if the bus k_s arriving from line s is assigned to be dispatched in position k_l of line l.
- Finally, set the corresponding dispatching time $d_{l,k_l}, \forall k_l \in B_l, \forall l \in L$. Let $h_{l,i}$ denote the operational headway for a specific line l as the time between the dispatching times of trips i and $i-1$.

3.2 Mathematical formulation of the MLVDP

Based on the above notation we now formulate the MLVDP as follow:

$$
\begin{aligned}
Q = \quad & \min_{\boldsymbol{d},\boldsymbol{h},\boldsymbol{x}} \sum_{l\in L}\sum_{i\in B_l}(h_{l,i}-h_l^*)^2, && (1)\\
s.t.\;\; & h_l^* \le \frac{H+d_{l,0}}{r_l}, \quad \forall l\in L, && (2)\\
& h_l^* \ge \frac{H+d_{l,0}}{1+r_l}, \quad \forall l\in L, && (3)\\
& d_{l,i} \ge f_{s,j}\cdot x_{(s,j)(l,i)}, \quad \forall l\in L, \forall i\in B_l, \forall s\in L, \forall j\in B_s, && (4)\\
& h_{l,i} = d_{l,i}-d_{l,i-1}, \quad \forall l\in L, i\in B_l, && (5)\\
& \textstyle\sum_{l\in L}\sum_{i\in B_l} x_{(l,i)(s,j)} = 1, \quad \forall s\in L, \forall j\in B_s, && (6)\\
& \textstyle\sum_{s\in L}\sum_{j\in B_s} x_{(l,i)(s,j)} = 1, \quad \forall l\in L, \forall i\in B_s, && (7)\\
& x_{(s,k_s)(l,k_l)} \in \{0,1\}, \quad \forall l\in L, \forall i\in B_l, \forall s\in L, \forall j\in B_s, && (8)\\
& d_{l,i}\ge 0, h_{l,i}\ge 0, \quad \forall l\in L, \forall i\in B_l, && (9)
\end{aligned}
$$

The objective of the problem (1) intends to dispatch at regular headways all buses included in the planning horizon for each of the lines that start their operations from the terminal. Thus, the objective function is to minimize the sum of the dispatching interval's deviation from the optimal dispatching headway of each line (h_l^*).

The set of constraints (2)-(3) determine for each line, the lower and upper bound of the optimal dispatching headway. Figure 2 shows the evolution of bus dispatches from time t onwards. The lower bound (2) is obtained by noting that all r_l buses must be dispatches during the period H. The first dispatch must occur $h_l^* - d_{l,0}$ unit of time after time t, while the following (r_l-1) buses needs to be dispatched every h_l^* units of time. The upper bound (3) establish that for a smooth transition between two consecutive operational periods (e.g., morning peak and off-peak period), the interval between the end of the period and the last dispatch must be at most the optimal headway h_l^*.

Constraint (4) determine the dispatching times for each bus in each line included in the planning horizon, while constraint (5) define the dispatching headway between two consecutive buses of the same line.

Constraint (6) ensures that every new trip $j\in B_s$ is executed by only one of any of the buses about to arrive at the terminal, which can be either of the same or a different line, while constraint (7) assigns an arriving bus $i\in B_l$ to a new trip and line. The nature of the variables is imposed in (8) and (9).

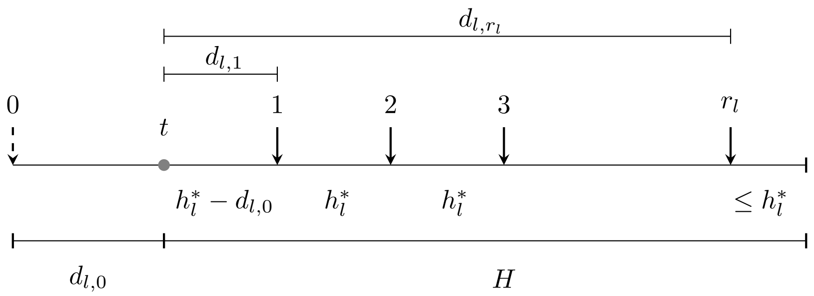


Figure 2: Dispatching dynamics.

Program Q is a Mixed-integer quadratic problem that simultaneously decides, for each of the lines that start their operations from the terminal their optimal dispatching headway. Also, for each of the buses included in the planning horizon, the MLVDP determines the dispatching time and the line to operate on its next trip.

3.3 Uniqueness of the solution and optimal dispatch policy

Given a feasible assignment $\hat{\boldsymbol{x}}$, problem Q reduces to find the bus dispatching times at minimum cost by solving the following problem:

$$
\begin{aligned}
Q(\hat{\boldsymbol{x}}, \boldsymbol{d}, \boldsymbol{h}) = \quad & \min_{\boldsymbol{d},\boldsymbol{h}} \quad g(\boldsymbol{h}) = \sum_{l\in L}\sum_{i\in B_l}(h_{l,i}-h_l^*)^2 \\
& s.t. \;\; h_l^* \leq \frac{H+d_{l,0}}{r_l}, \quad \forall l \in L \\
& \qquad h_l^* \geq \frac{H+d_{l,0}}{1+r_l}, \quad \forall l \in L, \\
& \qquad d_{l,i} \geq f_{s,j}\cdot \hat{x}_{(s,j)(l,i)}, \quad \forall l\in L, \forall i \in B_l, \forall s\in L, \forall j \in B_s, \\
& \qquad h_{l,i} = d_{l,i} - d_{l,i-1}, \quad \forall l \in L, i \in B_l, \\
& \qquad d_{l,i} \geq 0, h_{l,i} \geq 0, \quad \forall l\in L, \forall i \in B_l,
\end{aligned}
$$

Problem $Q(\hat{\boldsymbol{x}}, \boldsymbol{d}, \boldsymbol{h})$ has a quadratic objective function with linear set of constraints. Following the same procedure as in Gkiotsalitis and Van Berkum (2020) we would demostrate that problem $Q(\hat{\boldsymbol{x}}, \boldsymbol{d}, \boldsymbol{h})$ is convex.

Theorem. *Problem $Q(\hat{\boldsymbol{x}}, \boldsymbol{d}, \boldsymbol{h})$ is convex, thus every local minimum of the problem is also globally optimum (there may be more than one).*

Proof. The set of constraints of $Q(\hat{\boldsymbol{x}}, \boldsymbol{d}, \boldsymbol{h})$ are linear and thus the domain is a convex set. To prove that $g(\boldsymbol{h})$ is convex, we note that it can be expressed as $g(\boldsymbol{h}) = \sum_{l\in L} g_l(\boldsymbol{h})$ with $g_l(\boldsymbol{h}) = \sum_{i\in B_l}(h_{l,i}-h_l^*)^2, \forall l \in L$. If $g_l(\boldsymbol{h}), \forall l \in L$ is convex then $g(\boldsymbol{h})$ will also be convex, as it corresponds to the sum of convex functions. To investigate the convexity of $g_l(\boldsymbol{h})$, we will obtain its Hessian matrix $\boldsymbol{H}$. The Hessian matrix for line l with $|B_l|= n$, is:

$$
\boldsymbol{H} = \begin{bmatrix}
\frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_l^{*2}} & \frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_l^* \partial h_{l,1}} & \cdots & \frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_l^* \partial h_{l,n}} \\
\frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_{l,1} \partial h_l^*} & \frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_{l,1}^2} & \cdots & \frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_{l,1} \partial h_{l,n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_{l,n} \partial h_l^*} & \frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_{l,n} \partial h_{l,1}} & \cdots & \frac{\partial^2 g_l(\boldsymbol{h})}{\partial h_{l,n}^2}
\end{bmatrix} = \begin{bmatrix}
2n & -2 & \dots & -2 \\
-2 & 2 & \dots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-2 & 0 & \dots & 2
\end{bmatrix},
$$

which has eigenvalues of 0 and 2; therefore is positive semi-definitive and convex. □

Now, we present an small example that shows that problem $Q(\hat{\boldsymbol{x}}, \boldsymbol{d}, \boldsymbol{h})$ admits multiple solutions.

Consider a single line l where $H = 35$, $d_{l,0} = -5$ and $r_l = 4$. Also, $f_{l,1} = 1$, $f_{l,2} = 5$ y $f_{l,3} = 7$. According to constraints (2) and (3), $8 \leq h_l^* \leq 10$. If we set $h_l^* = 8$, then the following dispatching times $d_{l,1} = 3$, $d_{l,2} = 11$ and $d_{l,3} = 19$ ($h_{l,i} = 8,\ \forall i \in B_l$) are optimal with an objective function of 0. Note that this same objective function value is achievable if we take $h_l^* = 9$, and $d_{l,1} = 4$, $d_{l,2} = 13$ and $d_{l,3} = 22$ ($h_{l,i} = 9,\ \forall i \in B_l$).

3.3.1 FIFO dispatching discipline

In practice, bus operators prefer to assign buses according to a FIFO discipline. That is, assigning the first bus that arrives at the terminal to the first scheduled dispatch. In this way, we ensure a certain principle of justice between drivers by preventing a driver or bus that has arrived at the terminal earlier than another, ending up leaving later. This section introduces additional variables and constraints to our problem to ensure dispatches adhere to the FIFO discipline. Furthermore, we prove that a dispatch policy that follows this discipline is optimal for the original problem Q.

Let's define the following non-negative variable $z_{(s,j)(l,i)} = d_{(l,i)} \cdot x_{(s,j)(l,i)}$ which indicates the dispatching time of bus $i \in B_l$ that is about to finish its previous trip $j \in B_s$. Directly including this restriction in the model would result in a non-linear condition. To linearize it, we introduce the following set of constraints (10a) to (10d):

$$
\begin{aligned}
& z_{(s,j)(l,i)} \leq M \cdot x_{(s,j)(l,i)}, \quad \forall l \in L, \forall i \in B_l, \forall s \in L, \forall j \in B_s, && \text{(10a)}\\
& z_{(s,j)(l,i)} \leq d_{(l,i)}, \quad \forall l \in L, \forall i \in B_l, \forall s \in L, \forall j \in B_s, && \text{(10b)}\\
& z_{(s,j)(l,i)} \geq d_{(l,i)} - (1 - x_{(s,j)(l,i)}) \cdot M, \quad \forall l \in L, \forall i \in B_l, \forall s \in L, \forall j \in B_s, && \text{(10c)}\\
& z_{(s,j)(l,i)} \geq 0, \forall l \in L, \forall i \in B_l, \forall s \in L, \forall j \in B_s, && \text{(10d)}
\end{aligned}
$$

where M represents an upper bound for the dispatching time.

The FIFO dispatching rule is achieved by imposing the following set of constraints for all pair of buses $j_1 \in B_{s_1}$ and $j_2 \in B_{s_2}$ such that $f_{(s_1,j_1)} \leq f_{(s_2,j_2)}$:

$$\sum_{l\in L}\sum_{i\in B_l} z_{(s_1,j_1)(l,i)} - \sum_{l\in L}\sum_{i\in B_l} z_{(s_2,j_2)(l,i)} \leq 0, \tag{11}$$

Constraint (11) guarantees that if bus $j_1 \in B_{s_1}$ arrives before to the terminal than bus $j_2 \in B_{s_2}$, it will be dispatched earlier. Note, that $z_{(s,j)(l,i)} \geq 0$ only if $x_{(s,j)(l,i)} = 1$, otherwise $z_{(s,j)(l,i)} = 0$.

The MLVDP ensuring dispatches according to a FIFO discipline can be formulated as:

$$\begin{aligned}\bar{Q}: \ & \min_{\boldsymbol{d},\boldsymbol{h},\boldsymbol{x}} \sum_{l\in L}\sum_{i\in B_l}(h_{l,i} - h_l^*)^2, \\ s.t. \ & (2) - (9) \\ & (10a) - (11)\end{aligned}$$

In what follows, we will prove that and optimal solution to problem Q is also an optimal solution to problem $\bar{Q}$, i.e, a solution that complies with the FIFO discipline.

Theorem. *A bus assignment that satisfies the FIFO discipline is an optimal solution for problem Q.That is, the first bus to arrive at the terminal should be assigned to the earliest scheduled departure.*

Proof. Consider an optimal solution to problem Q that does not meet the FIFO discipline, as follows: $x_{(k,b_k)(i,b_i)} = 1$, $x_{(l,b_l)(j,b_j)} = 1$ where $d_{i,b_i} \geq d_{j,b_j}$ y $f_{k,b_k} \leq f_{l,b_l}$.

For the above solution to be feasible it must satisfy that:

$$d_{i,b_i} \geq f_{k,b_k} \tag{12a}$$

$$d_{j,b_j} \geq f_{l,b_l} \tag{12b}$$

Since $d_{j,b_j} \geq f_{l,b_l}$ and $f_{l,b_l} \geq f_{k,b_k}$ then $d_{j,b_j} \geq f_{k,b_k}$, and $x_{(k,b_k)(j,b_j)}$ is a feasible assignment.

Given that $d_{i,b_i} \geq d_{j,b_j}$ and $d_{j,b_j} \geq f_{l,b_l}$, then $d_{i,b_i} \geq f_{l,b_l}$ and $x_{(l,b_l)(i,b_i)}$ is a feasible assignment.

The new solution $x_{(k,b_k)(j,b_j)} = x_{(l,b_l)(i,b_i)} = 1$ is a feasible assignment to problem Q and satisfies the FIFO discipline. As the dispatching times remain unchanged, this solution is an optimal solution to Q. □

From now on we will work with problem Q.

3.3.2 Restrict the interchange between lines

Problem Q allows buses to freely interchange between lines. On some occasions, the interchange of buses with other lines is made impossible by restrictions in operation. For example, buses with specific characteristics (e.g., capacity) must be assigned to particular lines or preventing interchange between lines to keep drivers operating in a single service. On other occasions, bus exchanges between lines are executed only in exceptional situations, such as the lack of buses necessary to comply with the operational frequency.

Note that we can forbid the interchange between certain lines or between specific buses through the restrictions (6), (7). We can also discourage the interchange between lines by including in the objective function a term that penalizes this interchange, weighting it appropriately according to the operator's criteria. In this way, the problem Q becomes:

$$\begin{aligned}\tilde{Q}: \ & \min_{\boldsymbol{d},\boldsymbol{h},\boldsymbol{x}} \sum_{l\in L}\sum_{i\in B_l}(h_{l,i} - h_l^*)^2 + \sum_{s\in L}\sum_{j\in B_s}\pi_s \sum_{l\neq s}\sum_{i\in B_l} x_{(s,j)(l,i)}, \\ s.t. \ & (2) - (9)\end{aligned}$$

Where the term $\sum_{l\neq s}\sum_{i\in B_l} x_{(s,j)(l,i)}$ counts the number of interchanges between line s and all the other lines in L and π_s represents the weighting factor for line s.

3.4 Rolling Horizon dispatching policy

The MLVDP is a problem that dynamically decides the dispatching times and corresponding assignment of buses to each line. The underlying system faces significant operational uncertainties; for example, the remaining time for a bus to reach the terminal is not known precisely due to congestion, change in passenger demand that affects dwell times at stops, among others. To cope with this dynamic and stochastic environment, we adopt a rolling horizon policy (RH) (Powell, 2011). Every time a bus reaches the terminal, a rolling horizon policy estimates the future dispatching times and bus assignment decisions for all lines $l \in L$ and buses considered in the planning horizon (B_l). However, an RH policy will implement only the decision regarding the next dispatching bus. All other future decisions serve to estimate the system's future cost and therefore are discarded; the model will be solved again when a new bus arrives at the terminal where updated information of the system will be available.

4 Numerical experiments and results

In this section, we present three different experiments to answer the following questions. First, how does the length of the rolling horizon affects the quality of the dispatching decisions? Second, which is the value of flexibility? That is, how much can be gained by allowing buses to interchange between lines? Finally, how much savings are additionally obtained by having more accurate predictions of buses' remaining travel time to the terminal?

In the rest of this chapter, we will present the simulation environment, introduce the experimental design, and end this section by showing each experiment's results.

4.1 Simulation environment

We simulate our system using an event base and stochastic simulator. Every time a bus arrives at the terminal triggers a new event. In each event, we execute two main process: i) assign a bus to the next trip and determine its departure time and ii) update the state of the system, i.e. the remaining time until the arrival of each bus to the terminal ($f_{b,l}$), the number of buses (r_l) to dispatch before the end of the period of operation and the remaining time before the end of this period (H).

4.2 Experimental design

We define an scenario based on the number of lines that start their service from the terminal and their characteristics: the nominal frequency (fr_l) and the variability of the headway between the buses that arrive at the terminal represented by the coefficient of variation (COV). For each line, at the beginning of each simulation, we also know the number of buses that remain to be dispatched before the end of the operational period and the last bus's dispatching time.

The scenario under study, is inspired by the current operation of Terminal San José from the Transantiago system in Santiago, Chile. This terminal is located at the south of the city and serves as a maintenance and shelter for around 150 buses (Subus, 2021). Four different lines (201, 271, 201e, 228) start their operations from this point with nominal frequencies during the morning peak hour of 10, 6, 15, and 15 buses per hour respectively. The operation at this terminal presents important operational challenges as depicted in Figure 3 which shows the COV at the beginning and at the end of the route for each of these lines during the morning peak period. Each dot represents the COV for a specific line and day. The COV was obtained from the quality service indicators reported by the Metropolitan Public Transport Directorate (DTPM, 2021) for all Tuesday, Wednesday, and Thursday of August 2019. From Figure 3 we first observe that buses are rarely dispatched at regular headways which is highlighted by a large proportion of COV larger than 0.5. Second, we notice how the COV deteriorates along the route, where more than 40% of the observations reach a COV higher than 1 at the end of the service.

Based on the above information we simulate the last thirty minutes before the operational period ends ($H = 0.5h.$). We consider four lines $L = \{l_1, l_2, l_3, l_4\}$, with a nominal frequency (fr_l) of 15 buses per hour for l_1 and l_2 and 10 buses per hour for l_3 and l_4, while the COV between the buses that arrive at the terminal is set to one for all lines. To establish the number of buses left to dispatch on each line (r_l), we consider an operation in which the frequency's compliance is delayed, compared to the nominal one, on average a 15%. That is, the number of buses left to dispatch on each line is given by $r_l = \lceil H \cdot fr_l \cdot (1+\epsilon) \rceil$ with $\epsilon \sim U(0, 0.3)$. Finally, for our base experiment we assume that the remaining time until the arrival of each bus to the terminal is known accurately. We will relax this assumption in experiment 3.

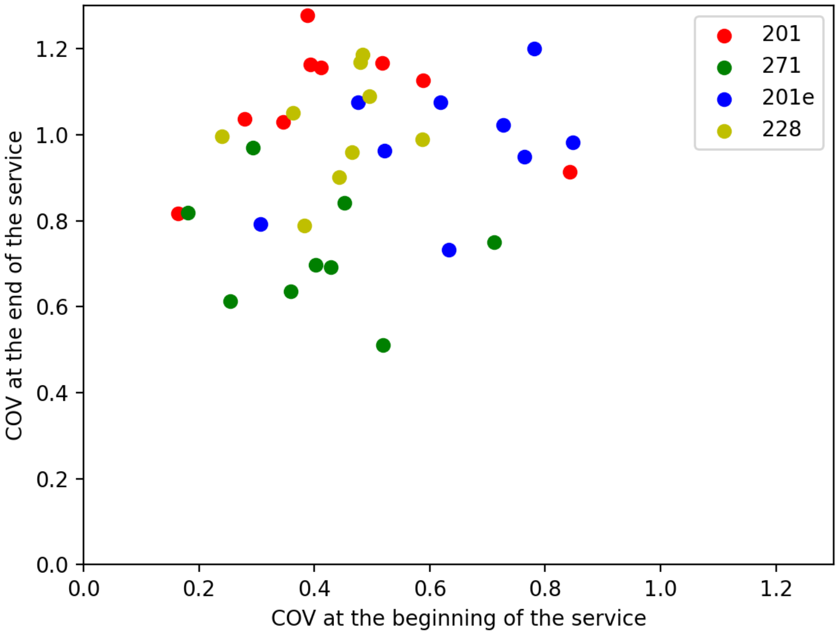


Figure 3: COV of headways at the beginning and end of the service for lines 201, 271, 201e and 228.

4.3 Results

We now present the results of all our experiments. We implement our simulation in Python 3.7 and use Gurobi 8.1 to solve the optimization models (MLVDP) every time a new bus arrives to the terminal. We run all our experiments on a computer with an Intel Core i5 2.6 GHz processor and 8 GB of RAM.

We computed the following performance indicators for each of the flexibility schemes and tested experiments: (1) COV of dispatch headways, computed as the average COV of dispatch headways considering all lines and (2) Frequency compliance, measured as the percentage of bus dispatches made compared to those required. To obtain each performance indicator we run 100 simulations for each case under study.

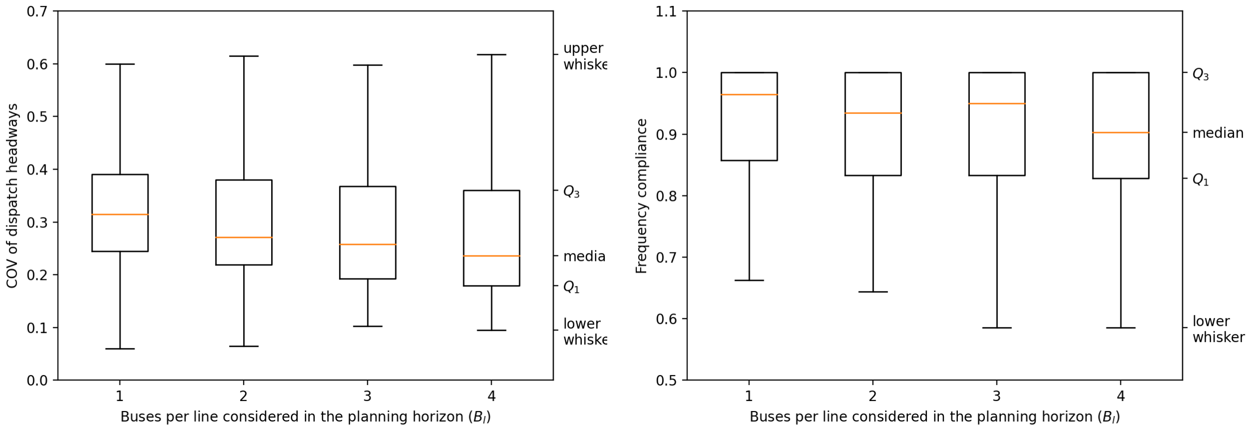
Experiment 1: Planning Horizon

Figure 4a and 4b presents the boxplot of the two performance indicators under different number of buses per line considered in the planning horizon. From Figure 4a we observe that the median COV of dispatch headways tends to decrease as the number of buses considered in the planning horizon increases. For the case in which we consider four buses in the planning horizon, this indicator's median turns out to be 33%, 15%, and 9% lower than when considering one, two, and three buses, respectively. Also, the variability of the indicator remains more or less constant across all the cases. The above suggest that increasing the number of buses in the planning horizon improves the regularity of dispatches.

Figure 4b shows that the best results in terms of frequency compliance are achieved for 1 and 3 buses in the planning horizon, wherein more than 50% of the simulated cases achieved compliance of more than 95%. This percentage drops to 90% when four buses are considered on the planning horizon.

Finally, Figure 5 presents the average computational time to take each dispatching decision. We observe an exponential grow in this indicator as the number of buses considered on the planning horizon increases. While, the average computational time for the case of one bus is on average 0.008 s. this number raises to 1.82 s. when four buses are considered.

Given the above results regarding COV of dispatch headways, frequency compliance and considering that decisions must be made in real-time, we set for the rest of our experiments a rolling horizon of $|B_l| = 3, \forall l \in L$.



(a) COV of dispatch headways

(b) Frequency compliance

Figure 4: Performance of the MLVDP under different number of buses per line considered in the planning horizon (100 simulations per case)

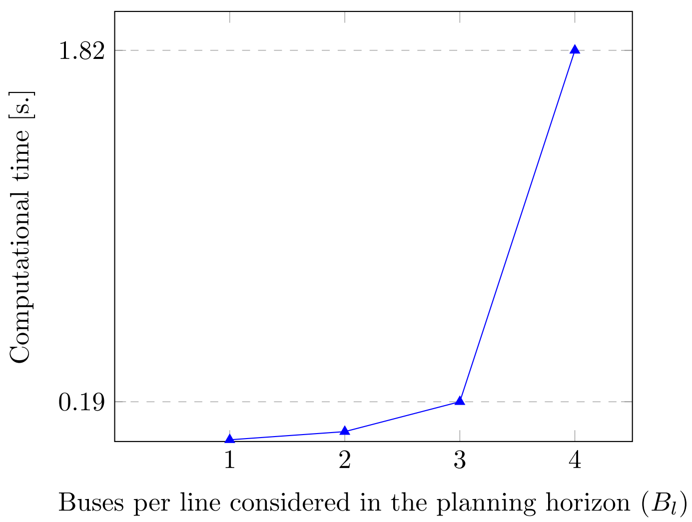


Figure 5: Computational time of the MLVDP under different number of buses per line considered in the planning horizon (Average over 100 runs)

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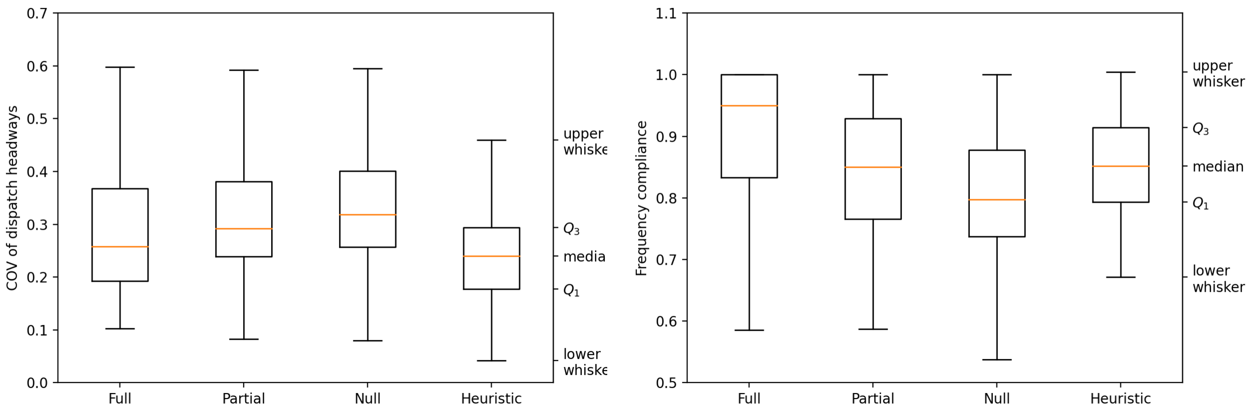
Experiment 2: Flexibility schemes

To quantify how much can be gained by allowing buses to interchange between lines, we compare three different flexibility schemes:

- *Null flexibility*: where each bus operates on a specific line without the possibility of interchange. This scheme is equivalent to have a set of fictitious terminals where a single line begins in each one. For each terminal, we run the Q model to determine the dispatch times of each bus in its corresponding line.
- *Partial flexibility*: where interchange is limited between subset of lines. This setting is equivalent to have a set of terminals, each one serving a group of lines. Thus, we solve problem Q for each terminal to determine the dispatching times of the lines that start at each fictitious terminal. In our experiment, we allow the interchange of buses between l_1 and l_2 and between l_3 and l_4.
- *Full flexibility* : where each arriving bus could be assigned to any of the lines that start at this terminal.

We also include a flexible *heuristic* dispatching rule for comparison purpose. The heuristic is based in a FIFO dispatching policy, it allows the interchange of buses between any line and adopts *a priori* target headway. Every time a bus b is ready to depart from the terminal, we compute for each line the interval between its last dispatch and the time when the arriving bus is ready for departure. If this interval is greater than the design (target) headway of the line, then the line is added to a candidate list. If more than one line is included in the candidate list, we assign bus b to the line with the greater deviation from the design headway, where the deviation is determined as the division between the computed interval and the design headway. In this case, the bus is dispatched to this line as soon it is available to depart. If none of the lines exhibit a computed interval greater than the design headway, then the bus is assigned to the same line it was operating. In this case, we hold bus b until the interval with the last dispatching bus of the line is equal to the design headway.



(a) COV of dispatch headways

(b) Frequency compliance

Figure 6: Performance of the MVLDP under different flexibility schemes (100 simulations per scheme)

Figures 6a and 6b present the results in the form of boxplot for the different flexibility schemes and the *heuristic* dispatching rule. Compared with the *Null flexibility* scheme, the median of the COV is reduced by 9% under the *Partial flexibility* scheme and by 19% compared with the *Full flexibility*, which shows how allowing more ductile operations can improve the regularity of the dispatch. Also, under the *Full flexibility* scheme, more than 25% of the simulation runs show a COV smaller than 0.2 while 75% show a COV smaller than 0.37.

We also observe the benefits of increasing flexibilibility when analyzing the frequency compliance. Again, the median frequency compliance under *Full flexibility* increases by 19% compared with *Null flexibility* and by 12% compared with the *Partial flexibility* scheme. Moreover, in 25% of the simulations, the *Full flexibility* scheme is able to dispatch all buses and in all simulations the frequency compliance is 83% or higher. In contrast, the *Partial* and *Null* flexibility scheme achieve only 50% of the simulations with a compliance higher than 85% and 80% respectively.

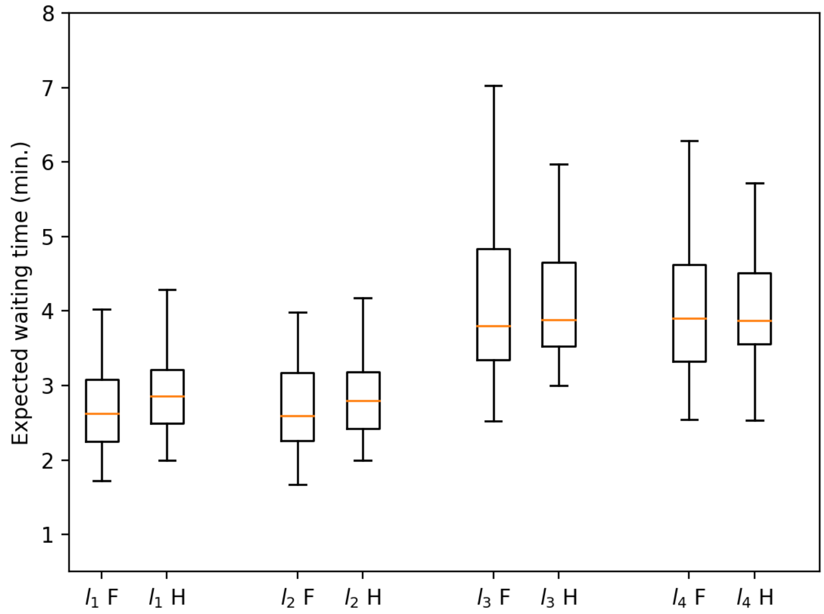


Figure 7: Expected waiting time under Full flexibility (F) and the Heuristic (H) scheme for the different lines (100 simulations per case)

If we now compare the *Full flexibility* scheme with the *heuristic*, we observe that the later is able to achieve more regular dispatches than the *Full flexibility* scheme. While the median COV for the *heuristic* yields a 0.24 this number rise to 0.26 for the *Full flexibility*, a 7.3% smaller. However, from Figure 6b we notice that the median frequency compliance of the *heuristic* is 11.5% smaller than that of the *Full* scheme. These results can be explained because the heuristic is only concerned with achieving regular dispatches according to *a priori* target headway without considering satisfying the operation's planned frequency. In contrast, the different flexibility schemes presented try to dynamically adapt dispatch intervals to achieve two objectives: meet the established operational frequency and maintaining regularity in the dispatch.

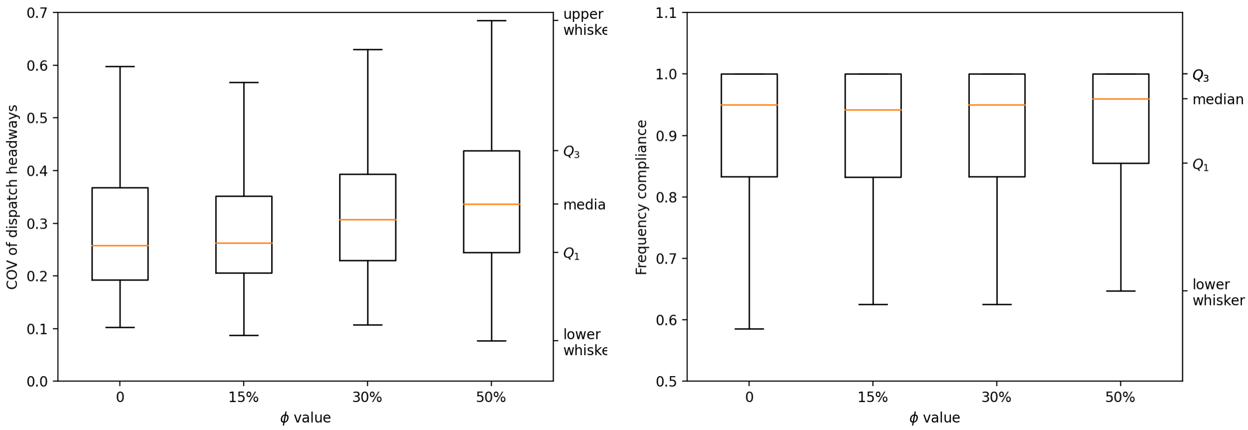
From the passenger's point of view, both the frequency and regularity of the headways directly impact waiting times. Assuming that the dispatch headways are maintained along the route, we can approximate the waiting time of each line according to the following expression (Welding, 1957; Osuna and Newell, 1972):

$$E[W] = \frac{E[h]}{2} \cdot (1 + COV^2) \tag{13}$$

Figure 7 depicts the box plot of the expected waiting time for each line for the *Full* and *heuristic* scheme. We can observe that the greater benefits of the *Full* scheme are obtained in lines with higher frequencies (l_1 and l_2), while in lines l_3 and l_4 the median waiting time are similar in the *Full* and *heuristic* scheme. We can explain the greater benefits obtained in high frequency lines because the difference in the number of buses to dispatch before the end of the period between the planned and the current operation may be greater in these scenarios. In our experiment, the expected number of extra buses to dispatch during H (30 minutes) is 1.125 for l_1 and l_2 and 0.75 for l_3 and l_4. Therefore, the *Full* scheme is capable to better adapt to this scenario where more buses must be dispatched. At the same time, the *heuristic* completely ignores this factor, only worrying about dispatching at regular intervals according to the target headway.

Experiment 3: Accuracy of travel time predictions

Our last experiment explores how much affect the performance of the MLVDP by having more accurate predictions of buses' remaining travel time to the terminal. To do so, we assume that the estimates of the remaining time to reach the terminal are obtained from a Normal distribution $f_{b,l} \sim N(\hat{f}_{b,l}, \sigma)$ with $\sigma = \hat{f}_{b,l} \cdot \phi$, where $\hat{f}_{b,l}$ corresponds to the precise remaining time for bus b from line l to reach the terminal.



(a) COV of dispatch headways

(b) Frequency compliance

Figure 8: Performance of the MLVDP under different levels of accuracy in remaining travel time estimations (100 simulations per case)

We consider three different values of $\phi = \{15\%, 30\%, 50\%\}$ and include for comparison purpose the case of perfect information ($f_{b,l} = \hat{f}_{b,l}$) which we will denote as $\phi = 0$.

From Figure 8a we observe that having more accurate information has a positive impact in reducing the COV of dispatched headways specially for ϕ values of 30% and 50%. The reduction achieved with perfect information translates that the median COVs are 20% and 30% lower than under the instances of $\phi = 30\%$ and 50%, respectively. It is important to note that mild errors in time estimations ($\phi = 15\%$) do not have important effects in this performance indicator reflecting the robustness of the model. We also observe from Figure 8b, that the frequency compliance is in general not affected by the accuracy of travel time estimations. The median in all cases is close to 95% while the 75% of the simulations runs independently of the ϕ value shows frequency compliance over an 83%.

5 Conclusions

In this article, we study what we call the Multiline Vehicle Dispatching Problem, which integrates the following decisions: decide the ideal dispatching headway for each line, assigns the line to operate, and determine the dispatching time for the following arriving buses to the terminal. The objective is to minimize the dispatching interval's deviation from an ideal headway that is dynamically updated based on the system's status. We present a Mixed-integer quadratic formulation and adopt a rolling horizon policy to cope with the dynamic and stochastic environment of public transit systems.

From the mathematical analysis carried out, we show that given an assignment of buses to operate, the problem of determining dispatch times from the terminal, admits (possible) multiple optimal solutions. We also prove that a bus assignment that satisfies the FIFO discipline is an optimal solution for the proposed problem. This result is essential for decision-makers because it provides an optimal policy to determine in which order to dispatch buses from a terminal.

We empirically test our model in a stochastic simulation environment under different operational conditions. The results demonstrate that the decisions times to take each solution are compatible with real-time applications. We also observe that allowing buses to interchange among lines reduces the COV of dispatch headways and improves frequency compliance by nearly 20% when compared with the case where buses are restricted to operate in a single line. Furthermore, our proposed MLVDP model shows to be superior in terms of expected waiting times, than a myopic heuristic scheme that allows full flexibility but considers *a priori* target headway. MLVDP demonstrate significant benefits in relation to the *heuristic* in high frequency lines in which the frequency's compliance is severe delayed compared to the nominal one. Finally, we found that our model is not very sensitive to mild errors in remaining travel time estimations to the terminal.

Avenues for future research include how to incorporate vehicle autonomy restrictions when assigning them to a line. In a context in which there is a fleet of electric buses, one can extend the MLVDP to integrate the decision to dispatch a bus to a new route or to send it to recharge its batteries.

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