MARKOVIAN DYNAMIC TRAFFIC ASSIGNMENT: A NEW APPROACH FOR STOCHASTIC DTA

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Keywords: stochastic DTA , Markovian , probabilistic route choice

ABSTRACT

We present the Markovian dynamic traffic assignment (MDTA) model, an integration of the Markovian traffic equilibrium (MTE) by Baillon & Cominetti (2008) with the dynamic traffic assignment (DTA) modelling framework by Addison & Heydecker (1996, 1998). It addresses stochasticity in a DTA context where motorists choose their route according to their perceived costs of the remaining trip, resulting in an arc-based choice model instead of a route-based one. Our model allows working with overlapping routes with no assumptions of independence of their costs. We also present the MDTA algorithm along with preliminar results regarding its computational implementation.

1. INTRODUCTION

In the context of traffic studies and transport planning, modelling motorists' route choices in a transport network through traffic assignment analysis is one of the most relevant issues when evaluating strategic and tactical transport investment projects. Static and deterministic formulations behind traffic assignment models are well established, with known properties and associated methods to calculate high-quality solutions efficiently. However, the use of static and deterministic formulations that assume steady-state networks without uncertainty precludes appropriate modelling when there is congestion in peak periods where overloaded networks cannot achieve steady-state conditions and/or when users have different cost perceptions.

Uncertainty in motorists' decisions has been addressed in literature by providing diverse stochasctic traffic assignment models and solution methods, mostly from a route-based perspective (such as Dial (1971)). This approach considers as the choice criterion users' perceived cost of travelling from their origins to their destinations. Approximately a decade ago, Baillon & Cominetti (2008) propose the concept of *Markovian traffic equilibrium* (*MTE*), an approach that differs from most of the others as its traffic assignment model considers that motorists choose according to the expected minimum cost from their current location to their destination. The MTE model overcomes the limitations of more straightforward stochastic models in its treatment of routes with common sections, as it is constructed as an arc-based model, rather than being a route-based one.

On the other hand, the dynamic formulation of the traffic assignment model has challenged researchers and remains a current topic, as it is a natural extension of static assignment models, in which routing decisions of motorists are assumed static over time. In the dynamic case, properties of capacity, causality, and flow propagation are features of difficult treatment, as it takes into account the time dependence of the demand in the assignment process. From the formal introduction of the *dynamic traffic assignment* problem (*DTA*) (Merchant & Nemhauser, 1978), DTA has been addressed through several approaches. Szeto & Wang (2011) proposed a DTA model classification that distinguishes: (1) the choice dimension; (2) the time dimension; and (3) the overall formulation approach. We have identified some groundbreaking articles, such as Addison & Heydecker (1996), that have crearly stipulated necessary requirements for the formulation of a suitable DTA model: a demand profile, a traffic model and a route-choice model. Comparisons of different traffic models to show how they could contribute to a DTA formulation, based on the pursued objectives, have also been addressed (Addison & Heydecker, 1998).

Traffic assignment literature has also addressed simultaneously time dependence and uncertainty, as stochastic versions of DTA models have been presented considering different ways to integrate uncertainty. Han (2003) and Szeto et al. (2011) present a route-based model and a cell-based model, respectively, with uncertainty in motorists' choices under dynamic assignment schemes. Han (2003) is an extension, to general networks and over discrete time, of a previous work by Heydecker & Addison (1997) where uncertainty comes from assuming that route costs are perceived differently by different motorists, and route choice is performed through a logit model with generalized cost as dominant criteria. Using simulation-based approaches, Long et al. (2019) and Barceló et al. (1999) incorporate stochasticity in the choice using simulated schemes to accomodate the dynamics behind the motorists' assignment, with fixed demand. Unlike most works, Waller & Ziliaskopoulos (2006) develope an analytic route-based model in a DTA context in which uncertainty is in demand. The approach in Fosgerau et al. (2013) approach generates an MTE model that could be interpreted as dynamic under certain considerations, such as deterministic costs and correction of utilities. Shimamoto & Kondo (2020) extended a static path flow estimation to a semi-dynamic version in a conveniently adjusted context.

The core contribution of this paper is the *Markovian dynamic traffic assignment* (MDTA) model, an analytical model for general transport networks that addresses time dependence in the demand and uncertainty in the choices of users, given by their different cost perceptions. Specifically, the arc-choice model proposed by Baillon & Cominetti (2008) in their MTE model is adapted to consider dynamic features following the modelling framework by Addison & Heydecker (1996). Another important contribution is the MDTA algorithm, a solution method inspired by *Dial's algorithm* (Dial, 1971) but repeated in each time increment and with a reversed scanning of the network in its two passes. We aslo explore some preliminar examples and sensitivity calculations to show the features of the solutions.

To get an intuitive idea of how our proposed Markovian approach impacts the assignment and differs from the usually used path-based models, let us consider Figure 1, where the demand from 1 to 3 will be assigned with a logit rule. From a route-choice approach, as the three paths from node 1 to node 3 have cost 1, the logit model assigns $1/3$ of the demand to each route, without taking into account that two of them overlap, but, under our arc-based approach, as the two lower routes differ from one another just at their ends, the solution assigns $1/2$ of the demand to the upper route and $1/4$ of the demand to both lower routes. On other aspect, unlike Fosgerau et al. (2013) or Shimamoto & Kondo (2020), our approach directly addresses the dynamic and stochastic aspects over general transport networks. Moreover, our model's treatment of overlapping routes is straightforward and needs no adjustment given the explicit arc-based formulations.

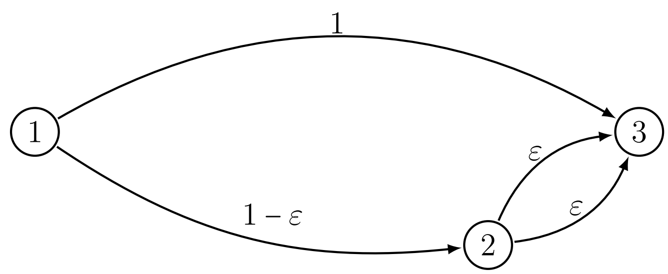

Figure 1: Simple network with constast costs (shown in each arc) to compare path-based versus arc-based assignments (Baillon & Cominetti, 2008).

Finally, we remark that the main goal of this paper is to propose a new approach to tackle the stochastic and dynamic dimensions in traffic assignment analysis by presenting the formulation of the MDTA model, proposing the MDTA algorithm as a solution method and showing some preliminar results regarding a computational implementations over an illustrative transport network.

2. MARKOVIAN DYNAMIC TRAFFIC ASSIGNMENT MODEL

We develop the MDTA model according to the modelling framework by Addison & Heydecker (1996). In this section, we introduce the concept of *reasonable arc* and then develop *the demand profile*, *the traffic model* and *the arc-choice model* (which serves as the route-choice model).

Let us consider a transport network represented by the digraph (N, A), where N is the set of nodes and $A \subseteq N \times N$ is the set of arcs; for each $i \in N$, A_i^+ and A_i^- are the sets of outgoing arcs from i and incoming arcs to i, respectively. For each arc $a \in A$, its free flow travel time ϕ_a and its queue unloading capacity Q_a are known parameters. Next, there are a set of origin nodes $O \subseteq N$, a set of destination nodes $D \subseteq N$, a set of O-D pairs $OD \subseteq O \times D$ and time-dependent demand rate functions from the origin to the destination of every O-D pair $(o, d) \in OD$, $\mathcal{D}_{(o,d)}(\cdot)$. The temporal horizon, represented by the time interval $[0, T]$, is also known.

2.1. Reasonable arcs towards a destination

In a pure stochastic concept, it could be assumed that each arc has a positive probability of being chosen, but reality, not all options are actually considered by motorists. Therefore, in a manageable and realistic model, a conveniently reduced set of arc options for users is desirable. With this purpose, Dial (1971) defines that, given an O-D pair (o, d), a route from node i to node j is a *reasonable route* for (o, d) if the minimum cost from o to i is less than the minimum cost from o to j and, simultaneously, the minimum cost from node i to d is greater than the minimum cost from j to d. Intuitively, this means that a reasonable route leads the motorist farther from his/her origin and closer to his/her destination if minimum cost routes are meant to be used.

We adapt the "reasonability" concept according to destinations (instead of O-D pairs) and over arcs (instead of routes). Given a destination node d, an arc (i, j) is a *reasonable arc* towards d if the minimum cost from j to d is less than or equal to the minimum cost from i to d. In this paper, we assume that a motorist decides wether an arc is reasonable towards his/her destination considering the arc costs when uncongested (equal to their free flow travel times). This establishes constant sets of reasonable arcs towards each destination d, denoted as R^d. This "reasonability" concept improves the complexity of our solution method (section 3), as it reduces the number of arcs to be considered when performing the assignment.

Next, we develop the three main structures of the MDTA model.

2.2. Demand profile

For each O-D pair $(o, d) \in OD$, the time-dependent demand rate function from the origin node o to the destination node d, $\mathcal{D}_{od}(\cdot)$, is given as this function is considered to be exogenous. These functions determine *the demand profile*, the first of the three parts of the model.

2.3. Traffic model

For the second structure, *the traffic model*, we adapt the *deterministic punctual queueing model* to represent the traffic behaviour within each arc, considering its features referenced in Addison & Heydecker (1998). Specifically, this part of the model characterizes, for each arc, the relationship between its inflow rates, outflow rates and queue lengths, and defines its cost function.

For each destination $d \in D$, for each arc $a \in A$ and at each time $t \in [0, T]$, the inflow rate and outflow rate of arc a going to destination d at time t are denoted as $E_{ad}(t)$ and $G_{ad}(t)$, respectively. The number of motorists with destination d in a queue on arc a at time t is denoted as $L_{ad}(t)$, which for simplicity we refer at as the *queue length going to* d of a at t (thus, the *total queue length* of arc a at time t, or *queue length* of arc a at time t, is $\sum_{d' \in D} L_{ad'}(t)$). There are two cases:

- The arc a is not congested: As the aggregated inflow rate does not surpass Q_a, no queue is formed, and the inflow rate going to each destination that enters the arc at t is able to exit the uncongested arc at $t + \phi_a$. This means that the outflow rate at $t + \phi_a$ going to each destination is equal to its respective inflow rate at t; and
- The arc a is congested: As the aggregated inflow rate surpasses Q_a, some motorists will not be able to leave the arc immediatly after traversing it. At $t + \phi_a$, the unloading is at capacity Q_a, splitting it proportionally according to the queue lengths going to each destination. Motorists that are not able to leave join the queue and experience a delay because of it.

Considering the introduced notation, the described behaviours for each destination node $d \in D$ for each arc $a \in A$ at each time $t \in [\phi_a, T + \phi_a]$ can be analytically expressed as:

$$G_{ad}(t) = \begin{cases} E_{ad}(t - \phi_a), & \text{if } \sum\limits_{d' \in D} E_{ad'}(t - \phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ \frac{L_{ad}(t)}{\sum\limits_{d' \in D} L_{ad'}(t)} Q_a, & \text{otherwise,} \end{cases} \tag{1}$$

$$\frac{dL_{ad}}{dt} = \begin{cases} 0, & \text{if } \sum\limits_{d' \in D} E_{ad'}(t - \phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ E_{ad}(t - \phi_a) - G_{ad}(t), & \text{otherwise.} \end{cases} \tag{2}$$

To determine the *cost function* of an arc (or, indistinctly, *total travel time*, *travel time* or *cost*), first, we need to consider that once entered an arc at a given time, all motorists experience the same cost, independent of their associated destinations. Now, for each arc $a \in A$ and at each time $t \in [0, T]$, the cost of the arc a, having entered it at t, denoted as $C_a(t)$, is given by the free flow travel time of a plus the delay due to the waiting time in the queue, if there is one. Analytically, we have:

$$C_a(t) = \phi_a + \frac{\sum\limits_{d' \in D} L_{ad'}(t + \phi_a)}{Q_a}. \tag{3}$$

Given the arc-based construction of the model, and particularly the construction of the arc costs, the condition of route cost independence, usually necessary in the DTA literature, does not have to be met. This states a defining property of our approach: being able to address overlapping routes without independence of route costs.

2.4. Arc-choice model

In the literature, route choices of motorists are usually built directly as a route-choice model. Under our arc-based approach, we represent these decisions through an arc-choice model. Thus, the recursive choices of arcs end up building the route that motorists travel through to go to their destinations. Our model is a dynamic adaptation of the static flow assignment embedded in the *MTE* concept (Baillon & Cominetti, 2008), which considers that motorists make their choice decisions following a logit model that considers the expected minimum costs from the current node to their respective destinations. We consider a constant and known dispersion parameter θ.

For each destination node $d \in D$, for each arc $a = (i, j) \in A$ and at each time $t \in [0, T]$, the expected minimum cost of going from i to d by choosing arc a, entering it at t, $Z_{ad}(t)$, is computed as:

$$Z_{ad}(t) = C_a(t) - \frac{1}{\theta} \ln \left(\sum_{b \in A_j^+} \exp\left(-\theta Z_{bd}\left(t + C_a\left(t\right)\right)\right) \right), \tag{4}$$

and the expected minimum cost of going from i to d, starting at t, $W_{id}(t)$, is given by:

$$W_{id}(t) = -\frac{1}{\theta} \ln \left(\sum_{a=(i,j) \in A_i^+} \exp\left(-\theta\left(C_a(t) + W_{jd}\left(t + C_a\left(t\right)\right)\right)\right) \right). \tag{5}$$

Therefore, from expressions (4) and (5), for each destination node $d \in D$, for each arc $a = (i, j) \in A$ and at each time $t \in [0, T]$, the following equations hold:

$$Z_{ad}(t) = C_a(t) + W_{jd}\left(t + C_a(t)\right), \text{ and} \tag{6}$$

$$W_{id}(t) = -\frac{1}{\theta} \ln \left(\sum_{a \in A_i^+} \exp\left(-\theta Z_{ad}(t)\right) \right). \tag{7}$$

Now, we can formulate equation for the inflow rates assignment. Let us recall that, because of the reasonability concept, given a destination d and a node i, only outgoing arcs a from i that are reasonable towards d (arcs a such that $a \in A_i^+ \cap R^d$, where R^d is the set of reasonable arcs defined in subsection 2.1) are assigned positive inflow rates; otherwise, no inflow rate is assigned. Additionally, at a given instant, the flow rate to be assigned from node i can come from two sources: the aggregate outflow rate of incoming arcs to i and/or demand rate being generated at i (if it is an origin).

Analytically, for each destination node $d \in D$, there are two cases: (1) for each node $i \in N$ such that $(i, d) \notin OD$ (nodes that are not origins for destination d), for each arc $a = (i, j) \in A_i^+$ and at each time $t \in [0, T]$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \begin{cases} \dfrac{\exp(-\theta Z_{ad}(t))}{\sum\limits_{b \in A_i^+ \cap R^d} \exp\left(-\theta Z_{bd}\left(t\right)\right)} \sum\limits_{b \in A_i^-} G_{bd}(t), & \text{if } a \in R^d, \\ 0, & \text{otherwise;} \end{cases} \tag{8}$$

and (2), for each $o \in O$ such that $(o,d) \in OD$ (nodes that are origins for destination d), for each arc $a = (o,j) \in A_o^+$ and at each time $t \in [0,T]$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \begin{cases} \dfrac{\exp(-\theta Z_{ad}(t))}{\sum\limits_{b \in A_o^+ \cap R^d} \exp\left(-\theta Z_{bd}\left(t\right)\right)} \left(\sum\limits_{b \in A_o^-} G_{bd}(t) + \mathcal{D}_{(o,d)}(t) \right), & \text{if } a \in R^d, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

This modelling framework defines the MDTA model. In section 3, a solution method is proposed.

3. MDTA ALGORITHM

In this section, we present the *MDTA algorithm*. This solution method works over a discretization of the analysed time period; thus, it delivers an approximate solution of the problem associated with the MDTA model. It is heavily motivated by *Dial's algorithm* (Dial, 1971) but with a reverse order of its steps, which are repeated in every time increment that results from the time discretization.

The inputs of the MDTA algorithm are the digraph (N,A) associated with the transport network; the set of origins $O \subseteq N$; the set of destinations $D \subseteq N$; the set of O-D pairs $OD \subseteq O \times D$; the free flow travel time ϕ_a and the queue unloading capacity Q_a of every arc $a \in A$, aggregated as the vectors ϕ and Q, respectively; the time-dependent rate demand functions for each O-D pair $(o,d) \in OD$, $\mathcal{D}_{(o,d)}(\cdot)$ aggregated as the vectorial function $\mathcal{D}(\cdot)$; the length of the time period, T; the timestep size Δt; and, for the logit model specifications, the dispersion parameter θ. For the time discretization, the number of intervals is set as $K = T/\Delta t$. Then, given $k \in \{1,...,K\}$, we refer to $[(k-1)\Delta t, k\Delta t]$ as the *time increment* k or k*-th time increment*.

The outputs are three arrays of size $|A| \times |D| \times K$, $E = \left(E_{ad}^k\right)_{a \in A, d \in D, k=1,...,K}$, $G = \left(G_{ad}^k\right)_{a \in A, d \in D, k=1,...,K}$, and $L = \left(L_{ad}^k\right)_{a \in A, d \in D, k=1,...,K}$. Here, given $a \in A$, $d \in D$ and $k \in \{1,...,K\}$, E_{ad}^k and G_{ad}^k are the inflow rate and outflow rate of arc a going to destination d at time increment k, respectively, and L_{ad}^k is the queue length going to destination d of arc a at time increment k.

The MDTA algorithm can be summarized in the following pseudocode:

Algorithm 1 (E,G,L) = MDTA$\left((N,A), O, D, OD, \phi, Q, \mathcal{D}(\cdot), T, \Delta t, \theta\right)$

STEP 0: INITIALIZATION Technical settings
for k=1,...,K **do**
  STEP 1: BACKWARD
  for all $d \in D$ **do**
    for all $i \in N$, in increasing order of mininum cost from i to d **do**
      for all $a \in A_i^-$ incoming arcs to i, **do**
        Compute expected minimum costs starting from i, through a, to d

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            end for
            Compute expected minimum costs starting from i to d
        end for
    end for
    STEP 2: ASSIGNMENT FACTORS COMPUTING
    STEP 3: FORWARD
    for all i ∈ N do
        for all d ∈ D do
            for all a ∈ A_i^+ outgoing arcs from i, do
                Compute the inflow rate, outflow rate and queue length going to d through a
            end for
        end for
    end for
    STEP 4: COST UPDATES
    for all a ∈ A do
        Update the cost of a because of the delays given by the current queue lengths
    end for
    STEP 5: STOP CONDITION
    if k = K or there are no more flow rates to assign then
        End MDTA algorithm
    end if
end for
```

Note that the MDTA algorithm allows initialization with non-empty transport networks. We do not address this feature further in this paper, but, given its potentialities, we intend to perform analysis in further stages of this research on some interesting cases where this property could become significant.

4. COMPUTATIONAL IMPLEMENTATION OF THE MDTA ALGORITHM

The MDTA algorithm has been implemented in MATLAB to test different instances. In this section, we present analyisis regarding one of those instances.

Let us consider the transport network with digraph (N, A), shown in Figure 2, where $(1, 13)$, $(2, 14)$ and $(4, 14)$ are the O-D pairs. For each arc a, the pair (ϕ_a, Q_a) shows its free flow travel time $[sec]$ and its queue unloading capacity $[veh/sec]$, respectively. The demand rate functions of each O-D pair, $\mathcal{D}_{(1,13)}(t)$, $\mathcal{D}_{(2,14)}(t)$ and $\mathcal{D}_{(4,14)}(t)$, are shown in Figure 3. We executed our algorithm over a period of T = 600 sec with a timestep size of Δt = 1 sec and a dispersion parameter of θ =0.2 sec^{-1}. For this case of a transport network of 14 nodes and 20 arcs, after repeared executions, the average running time is 32.79 sec.

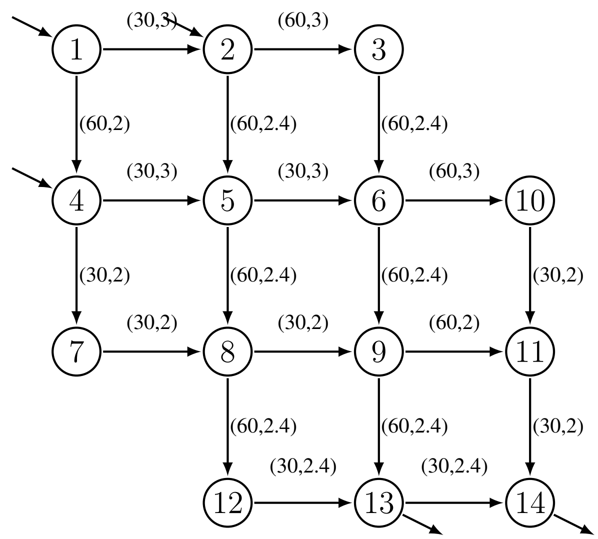

Figure 2: Network (N, A), with (ϕ_a, Q_a) on each arc a.

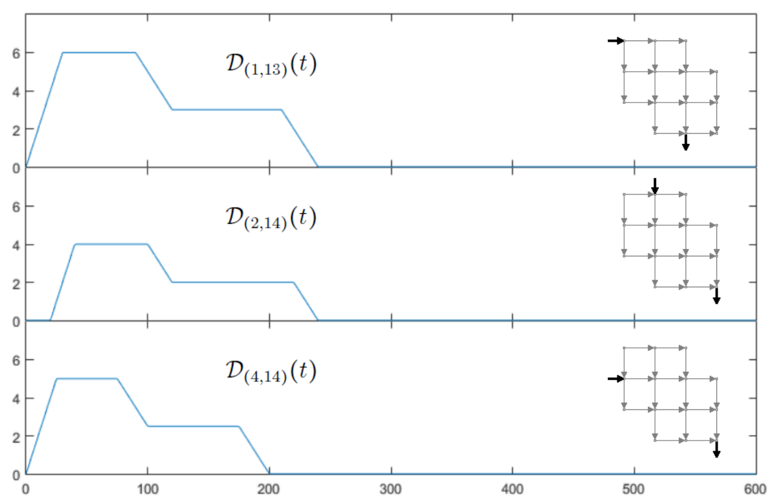

Figure 3: Demand rate functions $\mathcal{D}_{(1,13)}(t)$, $\mathcal{D}_{(2,14)}(t)$ and $\mathcal{D}_{(4,14)}(t)$.

4.1. General overview of outputs behaviour

Next, in Figure 4, we show the evolution of the outputs of arc $(4, 7)$. The following are important aspects to highlight the output behaviour:

- There are two types of flow rates: motorists going to node 13 (dashed blue lines) and motorists going to node 14 (dotted green lines). As they share the arc, their aggregation (continuous red lines) is affected by $Q_{(4,7)}$ (dashed light blue lines).

- When the total inflow rate surpasses $Q_{(4,7)}$; motorists reach the end of the arc after $\phi_{(4,7)}$ and, as not all of them are able to exit, some join the queue (or start one).

- When the queue length is positive, the total outflow rate is equal to $Q_{(4,7)}$, as the arc is congested and queue unloading happens at maximum capacity ($Q_{(4,7)}$).

- At a given time increment k, the total outflow rate equals $Q_{(4,7)}$ for one of two reasons: (1) there are no queues and the total inflow rate that entered earlier ($k - \phi_{(4,7)}$) is exactly equal to $Q_{(4,7)}$, or (2) there is a queue (as happens in the presented example). In the latter case, the total outflow rate (equal to $Q_{(4,7)}$) is split proportionally as outflow rates going to different destinations, according to the number of motorists waiting to leave the arc.

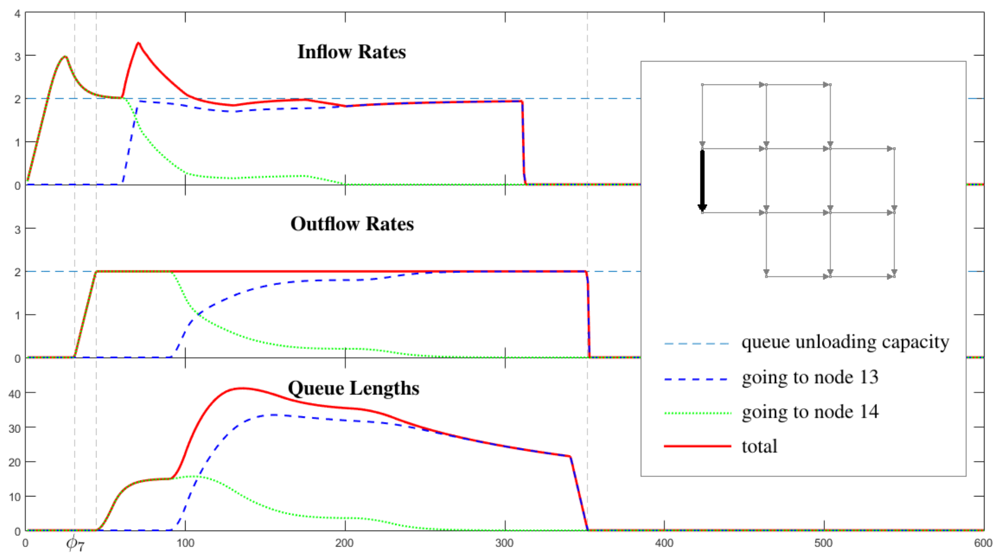

Figure 4: Evolution of outputs of arc 7.

4.2. Technical aspect: the dispersion parameter θ vs. the timestep size Δt

By observing the behaviour of the studied outputs, we note that, given certain combinations of values for the dispersion parameter θ and the timestep size Δt, the plots of the outputs (inflow rates, outflow rates and queue lengths) start showing a behaviour that cannot be interpreted as a realistic situation. This starts when the non-dimensional product of both factors is greater than 1.

To perform the analysis on our intuition, we compare the plots of the outputs that are obtained by fixing 6 different θ values: 0.1 sec^{-1}, 0.2 sec^{-1}, 0.5 sec^{-1}, 1 sec^{-1}, 2 sec^{-1} and 3 sec^{-1}, and the value of Δt needed to obtain the following $\theta\Delta t$ products: 0.8, 1, 1.2, 1.5, 2 and 3. In Figure 5, we show, for all these combinations, the plots of the inflow rate going to destination 13 of arc (1,2). We show only this arc because the conclusions are similar for the outputs of the rest of the arcs.

In Figure 5, we consider the plots of the column where $\theta\Delta t$ = 1 as a reference. We show just one value smaller than the reference to compare ($\theta\Delta t$ = 0.8), as all the other smaller values that were plotted show practically identical curves and acceptable behaviours compared to their respective reference, in the sense that there are no sudden changes in the flow assignment. The plots associated with higher values than the reference ($\theta\Delta t$ > 1) gradually start to show an oscillatory behaviour that cannot always be interpreted (as addressed later). First ($\theta\Delta t$ = 1.2 and 1.5), the plot exhibits minor oscillations (at the beginning of the peak) that dissipate in later time increments. Once started, these oscillatory behaviours do not dissipate and persevere over time ($\theta\Delta t$ = 2 and 3).

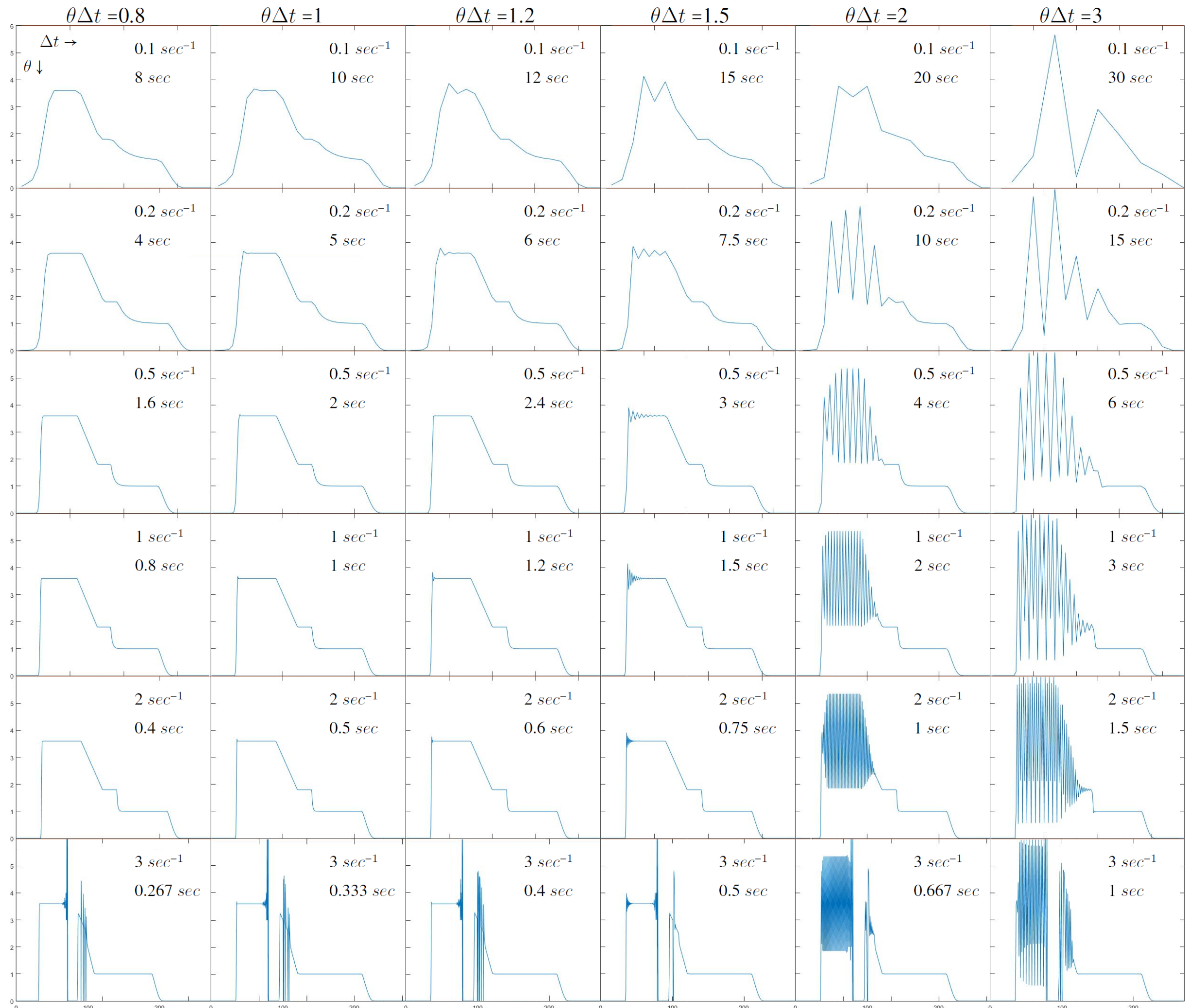

Figure 5: Evolution of the inflow rate going to node 13 of arc (1,2) under different combinations of dispersion parameter θ and timestep size Δt. In the upper right corner of each plot, the θ value (above) and the Δt value (below) are shown.

We can conclude that for θ and Δt values that lead to $\theta\Delta t$ slightly greater than 1, the plots of the outputs start to show some sort of oscillatory behaviour, which quickly scales to unrealistic outputs as $\theta\Delta t$ increases. This behaviour is questionable as it implies that motorists choose routes showing high sensitivity to cost changes. The more oscillatory the behaviour, the more difficult it is to explain, as from one time increment to the next one motorists choose to use one arc and then they do not, thus choosing the other one. On the other hand, plots where θ = 3 sec^{-1} present numerical errors that can be associated with the implementation of the algorithm and its numerical limitations.

5. FINAL REMARKS AND CONCLUSIONS

In this paper, we present the development of a novel model that allows facing a DTA problem under stochasticity in the motorists' decisions, namely, the MDTA model. The proposed approach comes from the integration of the contributions of two lines of work. The first is the traffic assignment that results from the MTE concept by Baillon & Cominetti (2008), initially applied for static cases. The second is the basis of the formulation presented by Addison & Heydecker (1996), who established that the DTA modelling process requires the development of a demand profile, a traffic model and a route-choice model. According to our model, the demand profile is exogenous, the traffic model adapts the deterministic punctual queueing model, and the route-choice model is built as an arc-choice model. The latter results from a dynamic extension of the MTE's traffic assignment.

Our main contribution is the MDTA model. The approach applies the notion that a motorist decides how to move forward considering the remaining part of his/her trip and does not decide according to his/her origin once he/she enters the transport network. To represent that, we introduce the *reasonable arc towards destinations* concept, which is an arc that, once travelled through, takes the motorist not farther from his/her destination if minimum cost routes are taken. Then, we assume that motorists travel through reasonable arcs only. The MDTA model has properties that are not usually found in DTA models from the literature. Given the arc-based approach rather than the usually assumed route-based approach, along with the within-arc interactions defined and formulated for the traffic model, the MDTA framework allows working with overlapping routes. This comes from the fact that route selection is actually a recursive arc-selection process. From applying this reasoning, independence on the route costs is not assumed, as the formulations are constructed according to the arcs. Thus, the only aspect regarding routing behaviour, which is the computation of the expected minimum costs from a current node to the destination experienced by the motorist, is constructed through nested arc cost operators. Additionally, route enumeration, usually applied to analyze and compare motorists' options, is not required. Also, even though the arc-choice model assigns the inflow rates according to the expected minimum costs through a logit rule, it is not limited only to this, as given the model construction, it has the potential of using different models to perform the assignment. The same can be concluded for the cost functions, where other models, apart from the deterministic punctual queueing model that we use in this paper, can potentially be used.

Another relevant contribution is the *MDTA* algorithm. The method allows obtaining an assignment for a discretized version of the problem and thus an approximated solution for the original version. In addition, the construction of the MDTA algorithm allows initialization with non-empty transport networks. From this feature, we can study how an already loaded network empties later on time if an MDTA approach is applied. Even though this property is not further developed in this stage of our research, it is a significant aspect to highlight, as it enforces the applicability of the MDTA model and its solution method. Our proposed method is a remarkable accomplishment, as traffic assignment solution methods are already complex to deal with and the MDTA algorithm is an efficient method that solves our proposed arc-based DTA approach through an elaborated dynamic programming routine, a defining achievement of this research. As for its computational implementation, we present, for a certain instance, the plots of different combinations of dispersion parameter θ and timestep size Δt values, observing that when the product $\theta \Delta t$ becomes greater than 1, the outputs start to present an oscillatory behaviour, scaling rapidly to unrealistic outputs. Furthermore, we observe that a dispersion parameter of θ = 3 sec^{-1} presents numerical errors, which we presume are related to the algorithm implementation. Therefore, we can conclude the importance of appropriately choosing the range of dispersion parameters for the logit model, which would lead to feasible interpretations, along with the appropriate size of the timestep in the discretization of the algorithm, that would lead to realistic implementations of the MDTA approach.

Before closing, it is worth remarking that the presented paper achieves what was established as the main goal: the development of a new traffic assignment model able tackle the stochastic and dynamic dimensions. Under the proposed approach, this research has been able to accomplish the contribution of the MDTA model, along with its proposed solution method, the MDTA algorithm. Among the potential research opportunities and extensions of the MDTA approach that we propose, we are especially interested in the following aspects:

- To study larger transport networks and to compare the resulting assignments and the ones that can be obtained from other models, particularly route-based;
- Response to incidents and capacity reductions, not all known in advance;
- Varying the concept of reasonability, particularly by extending it to the notion of an *expected set of reasonable arcs*;
- Use of different traffic models, such as Friesz' divided link model (Friesz et al., 1989);
- Potential application to the study of infrequent situations, such as gridlock. In this sense, a relevant article is Oyama & Hato (2017), where the authors address such situations by applying a sequential route-choice model according to expected utilities of the remaining part of the trip to the destinations.

ACKNOWLEDGEMENTS

This work was supported by projects ANID-PFCHA/Doctorado Nacional/2016-# 21161373, ANID/-FONDECYT/ REGULAR # 1191200, the Complex Engineering Systems Institute ANID PI-

A/APOYO AFB180003 and Research Regular Project code LPR19-11, VRIP, Universidad Tecnológica Metropolitana.

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