

VLT-LUT: MODELLING THE VERY LONG TERM EVOLUTION OF THE CITY IN 300 YEARS

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ABSTRACT

The evolution of cities is modelled to simulate the effects of agglomeration economies and transportation planning in the very long term (300 years or 1 to 29 million inhabitants). The VLT-LUT model simulates the evolution of land use and transportation and their interaction, based on urban microeconomic theory and market equilibrium. Preliminary results on an artificial city includes the impact of exogenous scenarios of road network evolution and agents' perception of agglomeration economies, observing the evolution of land use forms and the city size. Another result is that land rents evolve super-linearly with population, in line with previous research.

1. INTRODUCTION

In recent decades, complex models of cities development have been formulated and applied worldwide, specially for urban policies analysis, generically named *land use and transportation (LUT)* models (reviewed in Pagliara et al. (2010)). Such models share the approach of land use and transportation models, i.e., transfer information of transportation costs and location of activities. They aim to simulate city functions under different policy scenarios, usually in a time span of 20 to 30 years, through the interaction between representative households and firms agents and the movement of goods through the transport system, providing different performance indices that allow the planner to assess policy scenarios.

This medium-term perspective is useful for planning but lacks an understanding of the very long-term evolution. In this wider perspective, there is evidence of universal laws that emerge from worldwide urban big data (G. B. West, 1999; G. West et al., 1999, 2001), commonly observed in nature's complex systems (G. West, 2017). These are scaling laws with profound implications in the evolution of urban systems: they are superlinear (output per capita increases with population) i.e., despite differences in history (culture or geographical), cities follow a common law and evi-

dence shows that the universal law (power law) has a common scale parameter bounded to ± 0.15 (G. West, 2017). In contrast to this evidence, superlinearity is not approached by LUT models, as they are usually sets of microeconomic rules econometrically adjusted to each context. Conversely, a common universal law implies that parameters of the urban model are bounded by the scale parameter of the urban system.

To explain the scaling law, LUTE (Martínez, 2015) and CLUTE (Martínez, 2018) microeconomic models were formulated allowing the interaction between cities through demographic migration (geographically and socio-economically). CLUTE explains how the scaling law emerges from the complex interaction in the urban system, particularly, from individuals' and firms' rationality under uncertain information. With this theoretical foundation, the emergence of an urban science is conceived, as stated in Martínez (2018).

This paper reports -to our knowledge- the first attempt to simulate the evolution of cities in the very long term: a period of time of about 300 years or from 1 to 29 million inhabitants. It is designed as a research platform to analyze the different paths that a city may take in the very long term according to its agglomeration economies and transportation network, to extract lessons regarding the quality of life, productivity, and sustainability of the environment. We present a prototype model, the *very long-term land use and transportation (VLT-LUT)* software, a simplified version of LUTE and CLUTE models, where the economic submodel is replaced by accessibility indices and the geographical and socioeconomic migrations are exogenous submodels. We report preliminary results of the evolution of a fictitious city, simulated in a symmetric, flat, and homogenous plain, initialized with endogenously distributed population and firms and run for exogenous increments of population, keeping memory of the built infrastructure, and following an exogenous economic development measured by the city's GDP.

The aim of this paper, at this stage of our research, is not to replicate the evolution of a real city but to report on the performance of the VLT-LUT model under controlled scenarios. Given the symmetric conditions of the geography and the transportation network, we focused on analyzing whether the form of the complex urban system remains symmetric in the very long-term simulation, and how the scaling law is reproduced. The results of the reported simulations show a symmetrical evolution with different outcomes according to the respective behaviour of agents regarding two main factors: accessibility and agglomeration economies. We remark that values for factors and parameters that are used in this paper have been conveniently chosen, in order to be able to observe the behavior of the model while keeping the simplicity associated with the base scenario.

2. VERY LONG-TERM LAND USE AND TRANSPORTATION MODEL

In this section, we present the core contribution of this paper, the *very long-term land use and transport (VLT-LUT)* model, summarizing each submodel and setting some parameters. VLT-LUT represents the long-term evolution of an artificial city, considering a private transport network in a homogenous discrete land divided into 400 zones and the growth of agents, with residents categorized by socioeconomic attributes and firms by economic sector. The model is initialized inputting homogeneous allocation, including zone and building type, of 1 million agents in a city with a

given initial size and a given road network.

2.1. Land use model: CUBE Land

Agent's demand is modeled calculating agent h 's probability to be allocated in zone i and real estate type v , based on agents' bids for each location, computed as $\hat{B}_{hvi} = B_{hvi} + \xi$, where B_{hvi} is the deterministic component and ξ is a Gumbel error. The deterministic part is given by $B_{hvi} = b_h + b_{hvi} + b$, where: b_h is a reference bid that adjusts the utility levels to reach location equilibrium for each type of agent h (solved according to FPP3 below); b is a constant that adjusts bid levels to absolute price values in the economy; b_{hvi} is agents h 's value of zone i and real estate v . Because b_{hvi} includes the set of attributes describing neighbours' location externalities and economies of agglomeration, i.e., $b_{hvi}(P_{\cdot|i}, S_i)$, then the demand model solves fixed point problem ,FPP1 on auction allocation $P_{h|vi} = f(P_{\cdot|i})$:

$$P_{h|vi} = \frac{H_h \exp(\mu (b_h + b_{hvi}(P_{\cdot|i}, S_i)))}{\sum_g H_g \exp(\mu (b_g + b_{gvi}(P_{\cdot|i}, S_i)))}, \quad (\text{FPP1})$$

where $P_{h|vi}$ maximizes the stochastic bids simulating an auction, H_h is the total of agents of type h and S_{vi} is the supply of real estate type v in each zone i .

Rents r_{vi} result from the auction process and acquire the value of the maximum willingness to pay at each location and, given the Gumbel distribution of the bids errors, can be represented by equation 1 (endogenously supply-dependent, given construction's economy of scale):

$$r_{vi} = \frac{1}{\mu} \ln \left(\sum_h H_h \exp(\mu B_{hvi}) \right) \quad (1)$$

The land use model predicts the real estate market, estimating the supply S_{vi} for different types of real estate properties v in each zone i , for a situation with partial demolition of previous supply and based on a profit maximization Logit model. The model solves fixed point problem FPP2 on supply $S_{vi} = f(S_{\cdot})$:

$$S_{vi} = S_{0vi} (1 - k_{vi}) + (S - S_0 - S_D) \frac{\exp(\lambda \pi_{vi}(S_i))}{\sum_{w,j} \exp(\lambda \pi_{wj}(S_j))}, \quad (\text{FPP2})$$

with

$$S = \sum_{vi} S_{vi} = \sum_h H_h, \quad (2)$$

$$S_0 = \sum_{vi} S_{0vi}, \text{ and} \quad (3)$$

$$S_D = \sum_{vi} S_{0vi} k_{vi}, \quad (4)$$

where: S_{0vi} are supplies of the situation of the previous population level; k_{vi} is the demolition rate; π_{vi} is the expected value of developer profits $\hat{\pi}_{vi}$, defined as:

$$\hat{\pi}_{vi} = r_{vi} - C_{vi} + \varepsilon, \quad (5)$$

where cost functions C_{vi} are addressed later in this subsection (2.1.2), and ε is a Gumbel error.

The equilibrium between demand and supply models is reached assuming that all agents are allocated, which is attained by adjusting b_h variables of bids, solving the demand-supply equation $H_h = \sum_{vi} S_{vi} P_{h|vi}$, where $P_{h|vi}(b.)$, i.e., the equilibrium is a fixed point problem on b_h for all h (FFP3). CUBE Land model solves FFP3 and the result is the location matrix $H_{hvi} = S_{vi} P_{h|vi}$.

$$b_h = -\frac{1}{\mu} \ln \left(\sum_{vi} S_{vi} \exp(\mu (b_{hvi} - r_{vi})) \right). \quad (\text{FFP3})$$

2.1.1 Grid and variables

The grid defines 400 square zones of 25 [km^2]. Each zone i , $i = 1, \dots, 400$ represents a location option for any of the 5 types of agents, residential and non-residential, to locate any of the 5 types of housing (see Table 1).

type of agent h	characteristic	type of property v	characteristic
1	low-income home	1	small house
2	mid-income home	2	big house with yard
3	high-income home	3	apartment/office
4	Industry	4	commercial store
5	Commerce	5	large lot

Table 1: Types of agents and types of properties.

We define three types of attributes:

- Agents' attributes: The agents' variable is the income I_h , which is exogenous and defined only for residential agents ($h = 1, 2, 3$), with $I_1 = 13.283[UF]$, $I_2 = 25.618[UF]$, and $I_3 = 37.953[UF]$ (UF is a Chilean currency).

- Housing attributes: For each type of housing $v = 1, \dots, 5$, we define two attributes: building size q_c^v and land size q_t^v . The first one represents the size of the construction plan of housing type v and the second represents the land lot size used by v . The values assumed for these attributes are shown in Table 2, where: $v=1$ is a back-to-back house; $v=2$ is a detached house; $v=3$ is a flat; $v=4$ is an office; $v=5$ is an industrial property.

v	1	2	3	4	5
q_c^v	35	70	50	50	300
q_t^v	35	140	1	50	500

Table 2: Attributes of each type of property v [m^2].

- Zonal attributes: We define the residential, industrial, and commercial densities of zone i , ρ_i^{res} , ρ_i^{ind} , and ρ_i^{com} [hab/km^2], respectively, as:

$$\rho_i^{res} = \sum_{h=1}^3 \sum_{v=1}^5 H_{hvi} \frac{\theta_h}{A_i}, \quad (6)$$

$$\rho_i^{ind} = \sum_{v=1}^5 H_{4vi} \frac{1}{A_i}, \quad (7)$$

$$\rho_i^{com} = \sum_{v=1}^5 H_{5vi} \frac{1}{A_i}, \quad (8)$$

where A_i is the zone area ($25 [km^2]$), and θ_h is the average number of inhabitants by type of agent h . We use $\theta_h=3.5 [inhab/house]$ for $h = 1, 2, 3$. Densities represent a type of location externalities because they depend on the location of agents other than the bidder, thus introducing in the agent's bid B_{hvi} the location of others, i.e., bids depends on the location probability of all other agents and it is an endogenous variable in CUBE Land.

Access is also an endogenous variable in the LUT model, as it depends on densities and transportation costs. For each zone i , access is described by two components, *accessibility* to commercial activities (acc_i) and *attractiveness* to residents (att_i), defined as follows:

$$acc_i = \ln \left(\sum_{j=1, j \neq i}^{400} (\rho_j^{ind} + \rho_j^{com}) \exp(-\alpha_0 \tau_{ij}) \right), \quad (9)$$

$$att_i = \ln \left(\sum_{j=1, j \neq i}^{400} \rho_j^{res} \exp(-\alpha_0 \tau_{ji}) \right), \quad (10)$$

where, α_0 is a parameter equal to 0.03 min^{-1} , defining the disutility of τ_{ij} , assumed, while τ_{ij} represents the expected minimum travel time between the centroids of zones i and j (see subsection 2.2).

2.1.2 Cost functions

The cost function of housing type v in zone i , C_{vi} , is the construction cost plus the land lot cost, thus, we have that

$$C_{vi} = \alpha_1 q_c^v + p_i q_t^v, \quad (11)$$

with $\alpha_1 = 0.009 [UF/m^2]$.

Land prices $p_i [UF/m^2]$ are endogenous variables that depend on rents, while rents depend on supply S_{vi} , supply depends on the profits π_{vi} , and these depend on costs C_{vi} which depends on p_i , then $p_i = f(p_i)$ and can be expressed by fixed point problem FPP4.

$$p_i = \min_w p_{wi} = \min_w \frac{r_{wi}(p_i)}{q_t^v}. \quad (\text{FPP4})$$

2.1.3 Bid functions

We define agent h 's bid for property type v in zone i , b_{hvi} , as:

$$b_{hvi} = \begin{cases} \alpha_h I_h + \alpha_{1h} acc_i + \alpha_{2h} f_h(\rho_i^{res}) + \alpha_{3h} q_c^v + \alpha_{4h} q_t^v, & \text{if } h = 1, 2, 3, \\ \alpha_{1h} att_i + \alpha_{2h} \rho_i^{ind} + \alpha_{3h} q_c^v + \alpha_{4h} q_t^v, & \text{if } h = 4, \\ \alpha_{1h} att_i + \alpha_{2h} \rho_i^{com} + \alpha_{3h} q_c^v + \alpha_{4h} q_t^v, & \text{if } h = 5, \end{cases} \quad (12)$$

where values of α parameters are shown in Table 3. We also consider that the supply and demand Logit probabilities have scale parameters $\lambda = 0.03$ and $\mu = 2.5$, respectively. f_h represents the agent's perception of the density attribute, for which we consider four different shapes (Figure 1), where the base scenario is a linear function.

h	α_h	α_{1h}	α_{2h}	α_{3h}	α_{4h}
1	0.1	3.0	0.003	0.002	0.002
2	0.15	3.5	0.001	0.003	0.003
3	0.2	4.5	0.0005	0.004	0.004
4	-	3.0	0.01	0.01	0.05
5	-	4.5	0.002	0.05	0.01

Table 3: Parameters used in bids functions.

2.2. Transportation model: Markovian traffic equilibrium (MTE)

We first define the transportation network structure. To illustrate how, consider Figure 2 that depicts the 400 zones grid and a symmetric transport network (1 lane per direction roads) with a central

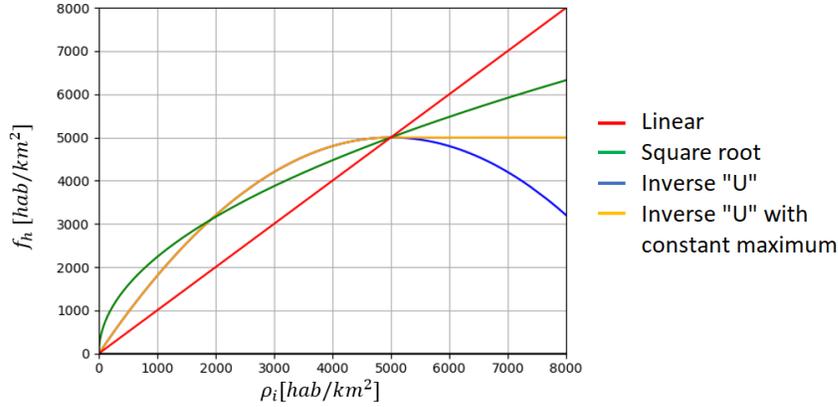


Figure 1: Different shapes of density perception function.

cross-shaped highway (2 lanes per direction). As shown in the inset of Figure 2, at each zone we define 17 nodes: a centroid (to receive and generate trips), 12 external (to interact with other zones); 4 internal (to interact with the centroid); 28 bidirectional arcs: 4 imaginary that connect internal nodes to the centroid; 24 real connecting non-centroid nodes, that can be of 1 or 2 lines.

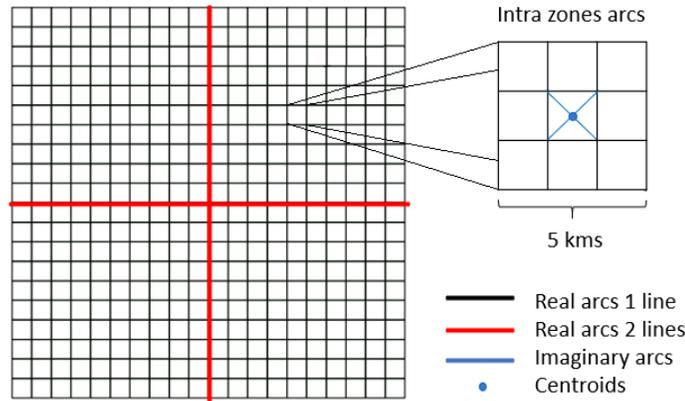


Figure 2: Structure of the network grid and zones.

We apply the Markovian traffic equilibrium (MTE) Baillon & Cominetti (2008), a traffic assignment model for private transportation where trips result from recursive arc choices. At a given node n and for a given destination node d , MTE assigns a portion P_a^d of the aggregated flow from all origins arriving at node n going to destination d , x_n^d , among each outgoing arc $a = (n, m) \in A_n^+$ (set of arcs leaving node n), by applying a Logit rule (with fixed dispersion parameter $\delta=1$) whose criterion is that travelers choose the arc with the expected minimum cost of going from n to d by using arc a , denoted as z_{ad} . Thus, we have that:

$$P_a^d = \frac{\exp(-\delta z_{ad})}{\sum_{b \in A_n^+} \exp(-\delta z_{bd})}. \quad (13)$$

Here, z_{ad} comes from the sum of the cost of arc a , t_a , and the expected minimum cost from m

(a 's end node) to destination node d , τ_{md} , thus $z_{ad} = t_a + \tau_{md}$. Note that expected minimum costs depend on the remaining trip to the destinations, i.e., $z_{ad}(\tau..)$, then, the model solves fixed point problem FPP5 on τ_{md} :

$$\tau_{md} = -\frac{1}{\delta} \ln \left(\sum_{a \in A_m^+} \exp(-\delta z_{ad}(\tau..)) \right). \quad (\text{FPP5})$$

On the other hand, the cost of arc a (t_a) depends on its flow w_a (equation 14),

$$t_a = \begin{cases} t_a^0 \left(1 + b_a \left(\frac{w_a}{c_a} \right)^{p_a} \right), & \text{if } w_a \leq c_a \\ 30 \text{ [min]}, & \text{otherwise,} \end{cases} \quad (14)$$

where: t_a^0 is the free-flow travel time; c_a is the capacity; $p_a = 3$ is a fixed non-linear parameter assumed $p_a=3$; b_a is a known parameter (values in Table 4). Variable w_a comes from the aggregation of flows of arc $a = (i, j)$ going to each destination d , denoted as v_a^d and computed as $v_a^d = x_i^d P_a^d$, thus, $w_a = \sum_d v_a^d$.

type of arc a	t_a^0 [min]	b_a [adimensional]	c_a [veh/h]
imaginary	0.1	0	1
real: bidirectional-1 lane	3.33	2	2000
real: bidirectional-2 lanes	2	4	4000

Table 4: Values for the congestion function parameters for every type of arc.

2.3. Demographic and firmographic models

The demographic and firmographic models estimate the population growth, number of agents, firms, and households. We replicate SECTRA's methodology SECTRA-MIDEPLAN (2003, 2008) to estimate the number of agents of each type h , H_h , $h = 1, \dots, 5$, for each forecasting year, based on the national estimate of GDP's growth per capita for mid-sized and large cities.

The demographic model estimates the total number of residential agents (types $h = 1, 2, 3$). For large cities, an estimation of income by decile is first computed, according to GDP, then, the probabilities of being on each decile range are computed to estimate each H_h . On the other hand, for mid-sized cities, the mean income of all agents is used to estimate H_h .

The firmographic model estimates the total number of firms (non-residential agents, types $h = 4, 5$). First, an estimation of the built surface of all types of use (industry, commerce, education, services, and others) is computed. With these results, H_h are estimated considering a linear dependency on the built area, with parameters that differ between large and mid-sized cities.

2.4. Trip generation and distribution model

The land use model's output provides the allocation of agents in the city, which is used to estimate trips between zones using private transportation costs provided by MTE model.

Given the land use, the trip generation and distribution (GDT) model seeks to estimate the daily movement of inhabitants. We use a standard doubly constrained entropy model to estimate trips between zones by solving a fixed point problem (FPP6), as in Macgill (1977). It uses as input the location matrix, the zonal densities, and the last computed travel times to deliver as output the number of trips between all pairs of zones. This obtained demand serves as an input for the MTE model, which then updates the travel times.

3. SIMULATION PROCESS

We integrate CUBE Land, MTE, and GDT connected by input-output file of access. In general terms, the equilibrium of the LUT model is attained by solving fixed problems FPP1, FPP2, FPP3, and FPP4 in CUBE Land, FPP5 in the MTE transportation model, and a FPP6 in the GDT model (Figure 3).

3.1. Modeling the urban spatial border

As population grows the city sprawls and the city border has to be modeled. We assume that urban and rural agents compete for location in all 400 zones. This competition is modelled external to CUBE Land and, for each i , according to the auction, with the profit $\pi_i = \max_v (r_{vi} - C_{vi})$, and an exogenous agricultural rent R_A . There are two cases: if the profit is less than R_A , then i is assigned to agriculture, no homes or industries will locate there and it is considered out of the urban boundaries; otherwise, i is included in the set of urban zones.

This process allows agricultural zones to become urban, but the inverse is ruled out, thus avoiding rural zones within the urban limit. This allows the city to extend its urban limits accordingly with the growing population following Alonso (1964)'s rule to define the city boundary as an auction between rural and urban land use.

3.2. Solution algorithm

We propose a solution algorithm that can be summarized as follows:

- **Initialization: Iteration 0:** A population of $N= 1$ million is set and distributed by agent type, using the demographic model. Next, an exogenous and homogeneous distribution of agents among zones is applied to compute initial values for the variables using free-flow

travel times. The urban limit feature is applied to obtain the Iteration 0 of the algorithm. Then, accessibilities and attractiveness are computed according to equations 9 and 10.

- **Iterative process (LUT for $N=1,3,5,\dots,27,29$ million):** The algorithm performs the LUT model for the current population N , obtaining the output solutions. Given that the model holds memory of past buildings, the resulting supplies S_{vi} are used as inputs for the following iteration ($N = N + 2$). It is worth noting that the solution of the LUT model for each population level does not reach equilibrium, because the algorithm executes the LUT interaction twice for each population level.

Figure 3 summarizes how the algorithm proceeds, including running CUBE Land, MTE, and GDT models twice for each population N and the identification of the urban limit, which is denoted as Z^N .

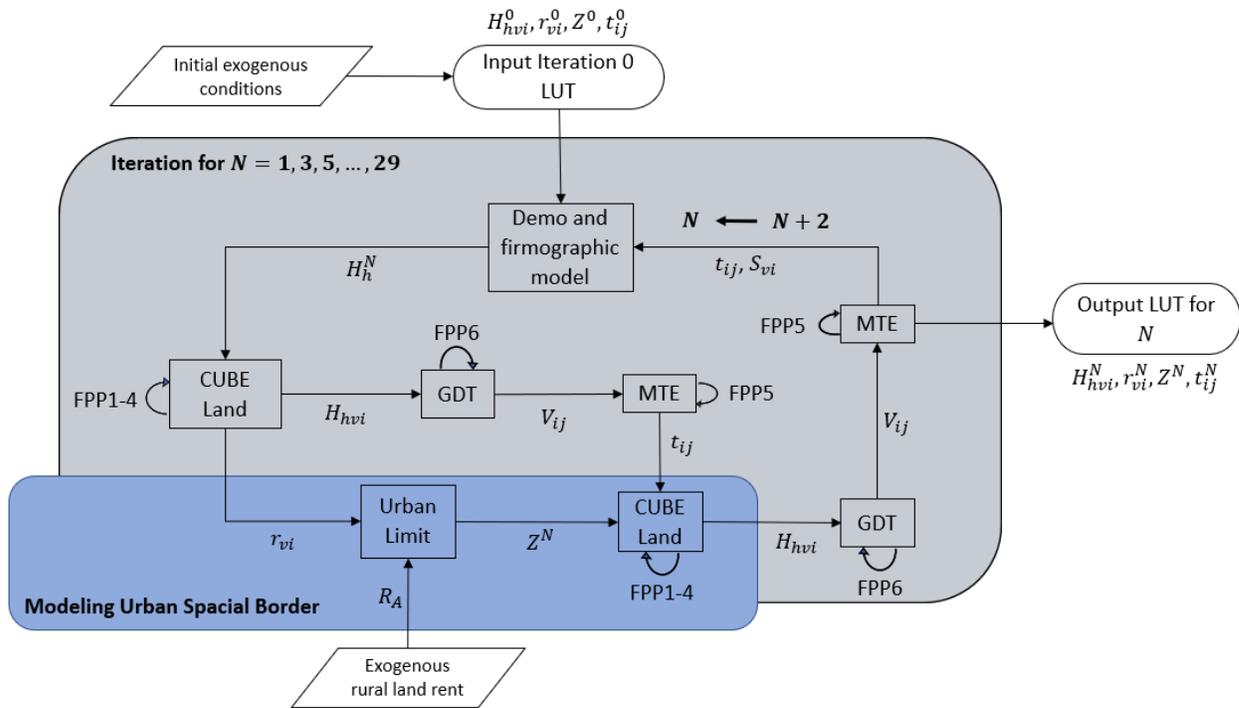


Figure 3: General algorithm scheme.

4. SIMULATION RESULTS

A computational implementation was carried out to develop a highly simplified fictional scenario, called the base scenario, as a proof of concept of the model, where the city evolves exclusively as a result of agents' behavior and market rules, using a linear bid function on densities (equation 12). Figure 4 shows the results obtained for the zonal variables in three population steps: 1, 15, and 29 million. We observe the following emerging urban forms:

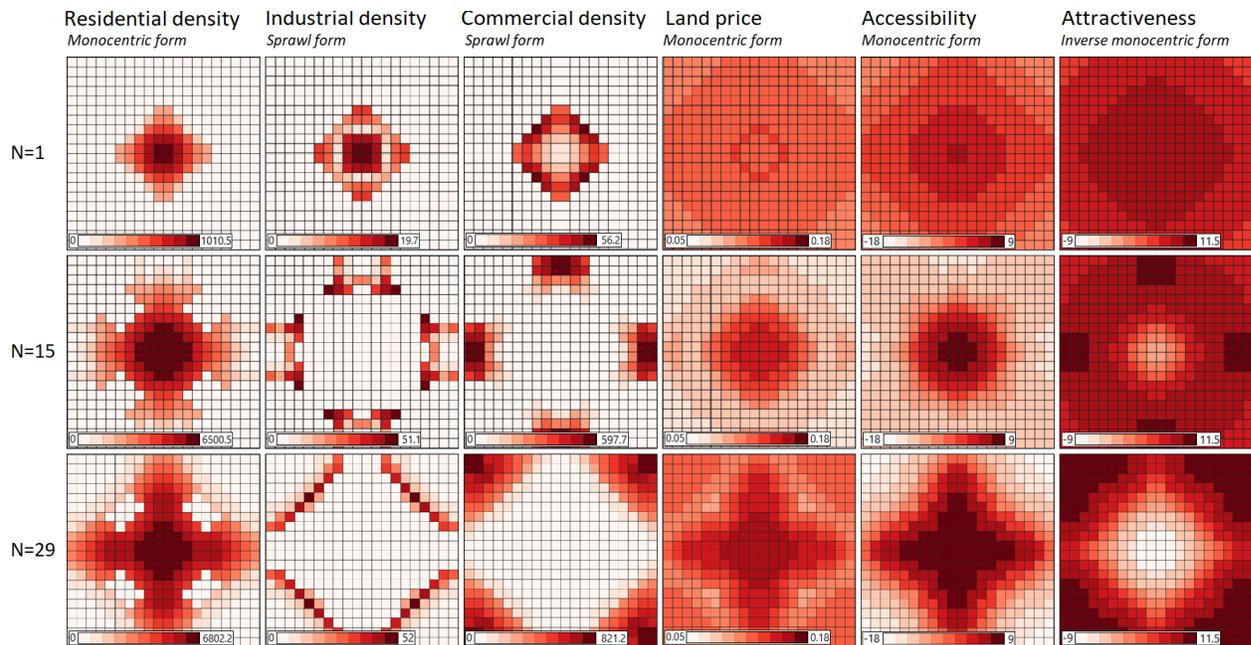


Figure 4: Evolution of attributes of the base scenario.

- i. Residential density grows from an initially monocentric shape to a cross shape following the transport highways. It depicts four rural (white) areas in each quarter's center (as rural use is more profitable to the landowner).
- ii. Industrial and commercial land use follows an extreme tendency to locate at the city outskirts, as an outcome of the high value set for bids parameters of these activities associated with agglomeration and land size (Table 3).
- iii. Land prices are highly monocentric and cross-shaped around the highway, but at $N=29$ million, the outskirts location of non-residential land use increase land prices of the corners of the grid, leaving lower rents in each quarter's center.
- iv. The spatial distribution of accessibility is monocentric combined with the cross shape of highways, while attractiveness is inverse monocentric.

A principal objective was to identify if the city evolves symmetrically despite the system complexity and the memory of the built area along with its population and size growth. As shown in Figure 4, symmetry is apparent, but we report that the system is highly sensitive to very small asymmetrical values (of the order of 10^{-10}) in the MTE estimations of travel time, such that the observed symmetry is obtained by rounding the model's values of trips to represent meaningful values of trips (vehicles per hour). Without rounding, the initial small asymmetry grows with population inducing asymmetric location patterns. Of course, symmetry is not a realistic feature of real cities, but it is a necessary property of the model under the fictitious conditions set in this scenario.

Additionally, Figure 8 depicts aggregate rents versus population in a logarithmic scale, showing that aggregate rents grow in a super-linear way with the population with a scale parameter of 1.09 approximately. This result demonstrates that the power law model $y = N^\beta$, supported by data G. West (2017) and explained by the theoretical microeconomic model Martínez (2018) implemented in the VLT-LUT software, is also replicated by our simulation results. It remains to analyze the effect on the scale parameters λ and μ of logit probabilities, because they are theoretically related Martínez (2018).

Then, three variations were made to the base scenario: case (1), changes in the transport network, where we increase the capacities of certain roads connected to the highways (Figure 5); case (2), changes in the bid function on densities, where we consider a superlinear function of densities followed by a saturation effect, i.e., after the saturation level, the density decreases its growth rate on bids functions (Inverse "U" in Figure 1); case (3), similarly to case (2), a superlinear function of densities is applied, but at saturation level its value remains constant (Inverse "U" with constant maximum in Figure 1).

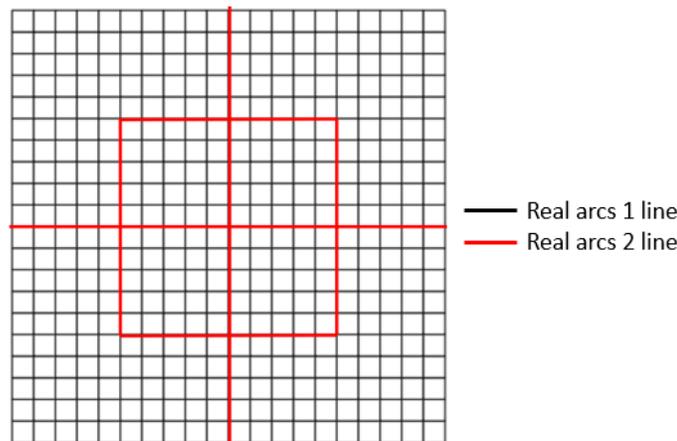


Figure 5: Structure of the network grid and zones of case (1).

First, regarding the attributes evolution, case (1) ends (at a population of 29 million) with practically identical distributions as the base scenario. Figure 6 shows the comparison of densities evolution in both cases. Note that they present a fairly similar evolution, i.e., the new transport network does not present a significant effect, moreover, rural zones do not change their use despite a greater road supply. On the other hand, case (2) and case (3) present substantial differences with the base scenario, as the growth of the urban special border is significantly faster, such that every zone becomes urban by the population level of 19 million inhabitants (in base scenario and case (1) there are remaining rural zones at the end of the simulation), while the allocation of agents is more homogeneous than those of the base scenario. Figure 7 shows the comparison of the densities evolution of case (2) and (3). Note that they start and end in a similar way, but they have a slightly different evolution in the urban spacial border.

Second, regarding the rent scale parameter, in all cases it decreases in comparison with base scenario: in case (1) to 1.07, close to the base scenario, in case (2) to 1.013, and in case (3) to a 1.014. It can be noticed that in both cases where the function of densities was modified, stronger decreases

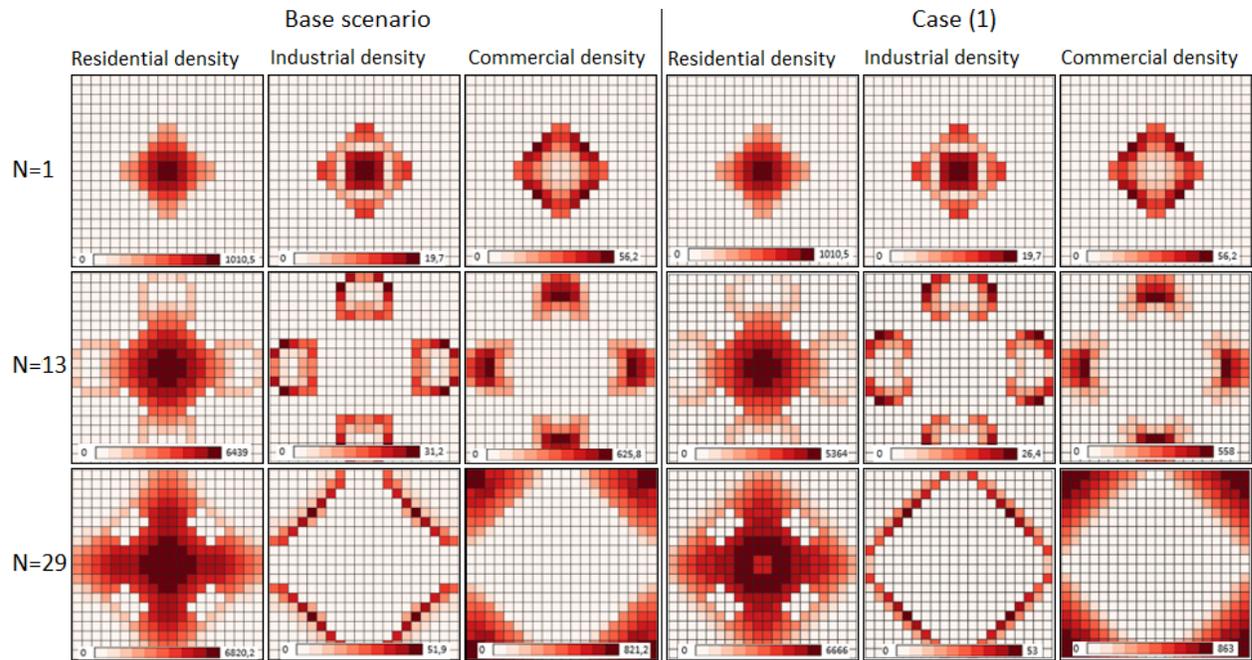


Figure 6: Comparison of the evolution of the density between base scenarios and case (1).

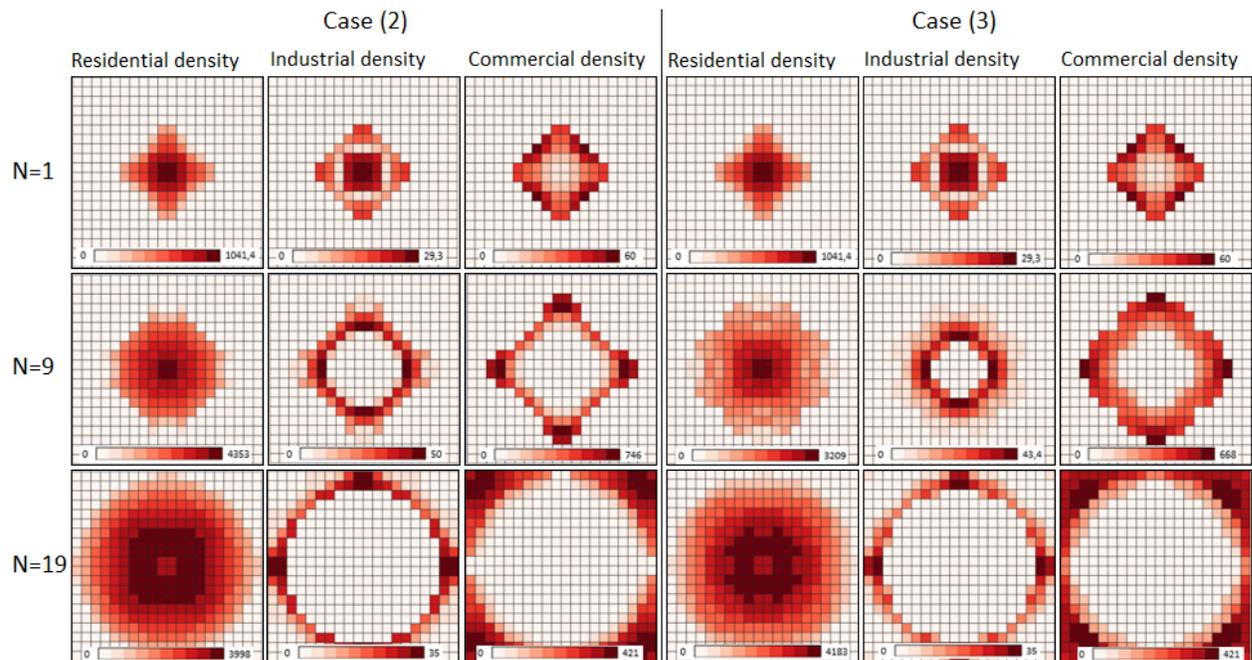


Figure 7: Comparison of the evolution of the density between case (2) and case (3).

affected the rent scale parameter. Figure 8 shows the scaling for the base scenario and for case (2) (case (3) is practically identical to case (2)).

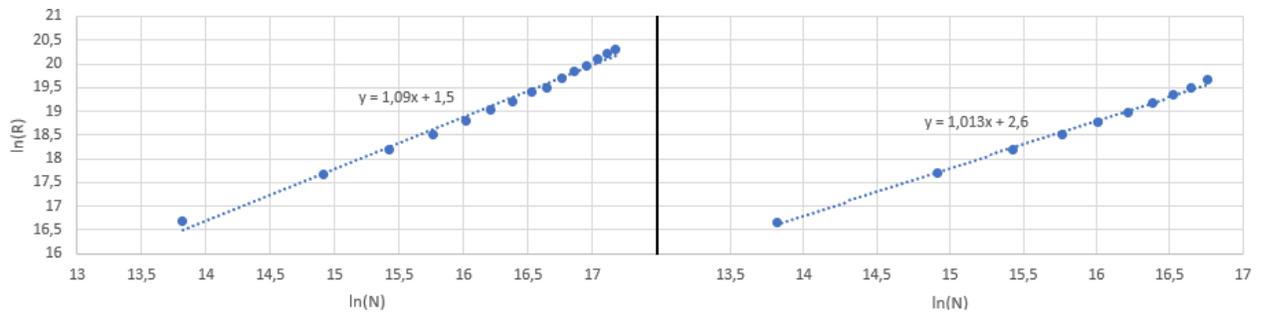


Figure 8: Scaling of aggregate rents for base scenario and case (2).

5. FINAL COMMENTS AND CONCLUSIONS

We conclude that the set agglomeration economies and transportation network parameters may induce spatial segregation of residential and non-residential activities. While the modeled scenarios are somehow unrealistically extreme, it is useful to observe the impact of the different sensitivities to agglomeration economies and transportation costs.

A second important conclusion is that the transport network shapes the city form. In the performed simulations, we let the transport network be highly simplified (private and with only one cross of highways) to emphasize how transport capacity induces location patterns. Thus, the urban form evolves from a small monocentric city to a shape that results from the combination of monocentric and transportation network shapes while agglomeration economies generate patterns of segregated agents. The emerging lesson is that roads infrastructure development is a policy that significantly shapes cities, while agglomeration economies are endogenous effects resulting from agents' behavior, which is exogenous to urban policies, although they may also be modified by subsidy policies.

It is important to highlight that this model is intrinsically dynamic, as it is not just a successive simulation of a city with different population levels, but a dynamically dependent process with memory (rural zones becoming residential, partial building demolition, and agglomeration patterns of specific agents).

Regarding future research, the plan is to simulate combined policies on zoning and road networks to obtain lessons about their long-term impacts and to analyze how the scaling law of rents is affected by them. A worthwhile extension of the software is to integrate, first, a more complex economic system with a labor market and, then, to integrate a system of cities in a region (as in Martínez (2018)).

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