**STRATEGIC DESIGN OF A TWO-TECHNOLOGIES TRANSIT NETWORK IN THE PARAMETRIC CITY MODEL**

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**ABSTRACT**

The *strategic design* of an urban transit system involves decisions regarding the structure of transit lines in space, the size of both fleet and vehicles of each line, their spacing, and their technologies. The many alternatives when considering two technologies is studied here using an approach based on the simple yet representative parametric city model PCM of Fielbaum et al. (2017), where meaningful initial designs play a key role. We present the main features of the PCM, a procedure to improve the initial designs, and three applications using Santiago-like parameters that generate interesting solutions.

*Key words: transit strategic design, two technologies, parametric city.*

# 1. INTRODUCTION

Investment decisions in urban public transportation have a long-lasting impact on the quality of life of the inhabitants. The *strategic design* of transit systems involves various dimensions: a transit network (the structure of transit lines in space), the size of both fleet and vehicles of each line in that network, their spacing (lines density), and their technology. Finding the best strategic design has to take into account all resources, including not only those aimed at producing capacity - operators' costs - but also those provided by the users, namely their time - users' costs.

In real-size networks, the strategic design of transit systems is an NP-hard problem that has been approached from different angles that are somehow complementary. One is of course the heuristic approach (e.g. Dubois, Bel, and Llibre, 1979; Ceder and Wilson, 1986; Cenek, 2010). A useful alternative are the analytical optimizing approaches, that rest upon two types of simplifications (Ceder, 2001): the representation of the city streets pattern by means of a regular network - a grid or circular schemes, as in Tirachini et al. (2010a), Daganzo (2010) or Badia et al. (2014) -, or the formulation of specific strategic problems by means of simple networks as in Gschwender et al. (2016) or Jara-Díaz et al. (2018). A different approach that combines regularity and simplicity while exhibiting better topological indices than the previous model types is the parametric city model (PCM) of Fielbaum et al. (2017), built upon the idea of centers that attract and generate trips. As explained below, most of these approaches have been developed considering one technology, usually buses. The objective of this paper is to construct and apply a method to obtain efficient strategic transit line structures in a city represented with sufficient generality using the PCM, admitting up to two technologies.

The simplest public transport structure to serve a given demand is the single circular line (Mohring, 1972; Jansson; 1980; Jara-Diaz and Gschwender 2003, 2009), which has been used to capture some structural design elements that have been shown to remain valid in more complex settings as well, such as the square root rule which indicates that optimal frequency and vehicle size growth proportionally to the square root of demand. Chang and Schonfeld (1991) include line spacing as a design variable in a model that considers parallel lines serving trips with uniformly distributed origins and a common destination at a single distant point. They obtain that the frequency of each line and the spacing between them are proportional to the cube root of the demand; frequencies increase less than in the case of an isolated line since line spacing decreases with demand volume; access time adjusts (diminishes) simultaneously with waiting time in such a way that their values are equal (see also Fielbaum et al, 2020).

The isolated line has been extended to simple and regular networks to explore the characteristics of certain line structures serving different spatial and demand conditions. Jara-Díaz et al. (2012) study the design of the spatial structure of transport services in a corridor when the demand is not homogeneously distributed, which was extended to consider two corridors that intersect in order to analyze the advantages and disadvantages of direct services as opposed to corridors with transfers. Gschwender et al. (2016) examined a similar model with dispersion at the extremes to explore the feeder-trunk structure. In parallel to the single line and simple networks, models based upon regular representations of cities, such as the grid model and the circular model, were developed as well. Daganzo (2010) analyzes three designs in a city idealized as a continuous grid over a square region: hub-and-spoke, grid lines, and a hybrid system composed of hub-and-spoke in the periphery and grid lines in the central area. Badia et al. (2014) extend the previous model to a continuous region of circular configuration considering two different scenarios in the distribution of trip origins -uniform and monotonically variable- and a hybrid network composed of a hub-and-spoke in the periphery and radial and circular lines in the central area. Motivated by the observed increase in circular lines in metro networks, Saidi et al. (2016) develop two analytical models, one that determines the number of lines in a radial scheme and another that focuses on planning and analyzing the feasibility to incorporate a circular line that complements and intersects radial lines. Badia et al. (2016) model a square city with two regions, a central and a peripheral one; the central area attracts all trips, which are generated in both regions with different densities. They consider four structures: direct lines, radial lines, grid lines, and a hybrid scheme combining grid lines in the central region and radial lines in the peripheral region. In the case of a city with a circular configuration, Badia (2020) analyzes three structures: direct lines, radial lines, and a hybrid scheme combining radial lines in the central region and hub-and-spoke in the peripheral region. Both models show that the best line structure is determined mainly by the size of the central region and not by the relative trip generation rates across regions.

Neither regular nor simple networks have topological indices that resemble those of real cities (Fielbaum et al, 2017), which has motivated the representation of cities based upon the relationship between centers and the minimum network for their interaction through a limited set of parameters, enough to describe the spatial structure and the demand pattern. This city model was built aiming at the strategic design of transit networks; it admits as particular cases the representation of monocentric, polycentric, and dispersed cities, and has been used to study the design of line structures considering a single mode (bus), where optimal line frequencies and vehicle sizes are calculated (Fielbaum et al, 2016). In particular, the center-based PCM has been used to analyze and compare preconceived well-known strategic one-technology transit networks as feeder-trunk, hub-and-spoke, direct (no-transfers), and exclusive (no-stops).

The optimization models synthesized above consider single-mode networks. Single-mode models have also contributed to inter-modal comparison involving operators and users costs for the analysis of different technologies, as in Allport (1981), who showed the important role of users' costs in the comparative analysis. Daganzo (2010) analyzes three technologies in isolation: bus, BRT, and subway in a regular square region with uniform demand and different network configurations. The results show that BRT outperforms the rail systems because the high infrastructure costs of rail preclude spatial coverage which cannot be compensated with high speed and capacity. Tirachini et al. (2010b) found that for a single line, buses and BRT are the most cost-effective modes for low and intermediate demand values, respectively. Heavy rail emerges as the best alternative only when BRT capacity is not enough to meet demand. These comparisons across modes pose a very natural question: why not use a combination of two technologies to find a set of lines that offer the best (social cost minimizing) design for a given transit demand? This challenge has been addressed in the literature in two directions: design along a corridor, and design on a regular network.

In corridors, Jara-Diaz and Tudela (1993) used a multi-objective approach to find the non-inferior combinations of price and frequency for a bus service that runs along a corridor feeding a subway at a given station. Chien and Schonfeld (1998) optimized simultaneously the design variables of feeder bus lines and a trunk rail line (e.g. headways, stops spacing). Sun et al. (2017) analyze a bus-train feeder-trunk structure to find the optimal rail length when demand is assumed to increase from the city boundary to the CBD as a single destination. Sivakumaran et al. (2012) also study a corridor where demand has a single destination, aiming at the design of frequencies of feeder and trunk vehicles while coordinating their arrivals in order to reduce total costs (operators' plus users'). Jara-Diaz and Muñoz-Paulsen (2021) depart from preconceived lines structures to examine all possible combinations to serve the demand along a corridor considering a periphery, a sub-center and a CBD; they find that the very popular feeder-trunk structure becomes appropriate only under very special demand distribution conditions.

Bi-modal systems have also been analyzed in regular networks, particularly on a rectangular grid, as done by Sivakumaran et al. (2014) and Fan et al. (2018). In both papers bus lines feed perpendicularly rail (or BRT) trunk lines in order to find the social cost minimizing design depending on demand densities and trip lengths. Results show that two-technologies networks (mainly bus-BRT) are better than a single-mode network for intermediate and high demand values and that joint design is superior to the separated design of local and express services. It is quite evident that the literature on bimodal structures has emphasized the feeder-trunk design (buses with rail or BRT) under very limited demand patterns, dismissing other design schemes and general demand conditions. The main challenges that are faced when conceiving the strategic design of an urban transit network considering the potential use of two technologies represented by their corresponding operators' costs (e.g. bus-subway), can be successfully faced using the PCM as it was conceived specifically for this, combining the advantages of a relatively simple while flexible representation of a city and its travel demand pattern with the capability to capture the main structural elements to build strategic transit networks.

In order to generate and apply a procedure to approach the design of appropriate line structures for a given city type, in the next section we present the main features of the PCM, followed by a recursive procedure to deal with the design of two-technologies networks. In Section 3 we present an application using three starting designs, one based upon a general lines structure and two other based on observed networks. Section four synthesizes and concludes.

## 2. TWO-TECHNOLOGIES STRATEGIC TRANSIT DESIGN USING THE PCM

### 2.1 The parametric city model (PCM) and transit lines typology.

Following Daganzo (2010), Badia et al. (2014), Badia et al. (2016), Badia (2020), and Fielbaum et al. (2016), the strategic design of an urban transit system involves a set of lines - their itineraries, frequencies, and vehicle sizes - organized in such a way that all trips can be served. In order to find the most adequate structural elements, an appropriate topological description of the city and its trip structure - an Origin-Destination matrix or a pattern of travel densities - is required. Here we will follow the representation proposed by Fielbaum et al. (2017) - which we call the parametric city model (PCM) - specifically conceived to deal with the normative analysis of transport systems; it is based upon the concept of centers: a CBD and $n$ zones, each with a periphery (P) and a sub-center (SC). The PCM has geometric and demand parameters, as shown in Figure 1; the former includes the number of zones and distances (P-SC, SC-CBD, SC-SC), and the latter includes total demand and generation-attraction parameters, which can be chosen to represent a meaningful period for design, e.g. a simplified morning peak period assuming that peripheries only generate trips, that the CBD only attracts; and that the subcenters do both. As evident, this representation presumes an underlying location and land use pattern. Within this setting, $Y$ is the total demand per hour, $a$ is the proportion of trips that start at peripheries, out of which a proportion $\alpha$goes to the CBD, $\beta$to the own subcenter, and $\gamma$ to the other subcenters, such that $\alpha + \beta + \gamma = 1$. A proportion of trips $b = 1 - a$ starts at subcenters and goes to the CBD and to other subcenters in proportions $\tilde{\alpha} = \alpha/(1 - \beta)$ and $\tilde{\gamma} = \gamma/(1 - \beta)$, respectively. In this way, the symmetric version of the city is defined with three geometric parameters $(n, g, L)$ and four demand parameters $(Y, a, \alpha, \beta)$. As defined, the parameters admit the representation of the main city types: monocentric, polycentric, and dispersed, when $\alpha$, $\beta$, or $\gamma$ tend to 1 respectively. The graph implies the aggregation of parallel streets in each arc, usually part of the main road network of the cities. This can be modified to admit parallel transit lines (lines spacing or its inverse, lines density) and treated either as a design variable (as in Fielbaum et al, 2020) or as an additional parameter, such that lines run along different $D$ parallel streets (or ways), depending on the infrastructure related to their vehicle technologies.

Although the path followed by a transit line can be generically described by the sequence of nodes visited, the PCM facilitates the description of some basic or regular lines using geometric concepts such as diametric (those that cross through the CBD), radial (those that reach the CBD) or circular (those that visit all subcenters). This, in turn, makes it easier to describe some well-known strategic line structures.

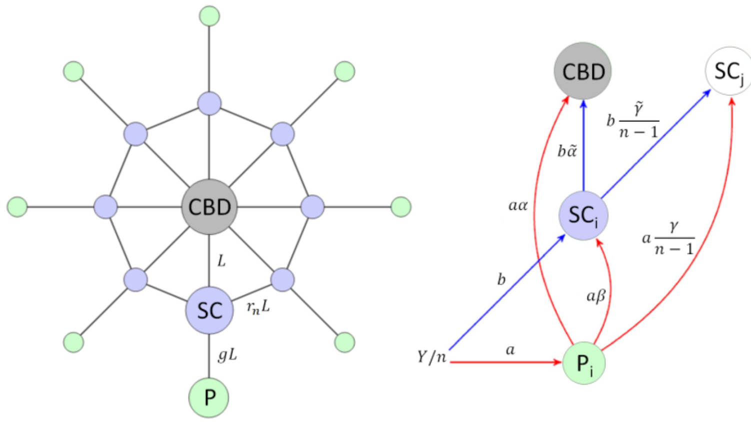


**Figure 1. The PCM: topological representation (symmetric case) and demand structure**

Following Figure 2, let us describe five types of regular lines. Circular lines (Figure 2a) connect the ring of subcenters, which can be clockwise and counterclockwise. Zonal lines (Figure 2b) connect the peripheries with their own subcenters. Radial lines (Figure 2c) connect each zone with the CBD; depending on whether the line starts at a periphery or a subcenter, they can be sub classified as radial-long (red line in Figure 2c) or radial-short (blue line), respectively. Diametric lines connect two zones through the CBD separated up to $n/2$ or $(n-1)/2$ zones (with $n$ even or odd respectively), admitting the same sub-classification of short and long as the radial ones; Figure 2d represents an example considering a separation of three zones (3z). Tangential lines connect two zones through a section of the ring of subcenters, usually distanced up to $\lceil n/4 \rceil$ zones, as in the example of Figure 2e considering a separation of two zones (2z); these lines can also be subclassified as long and short.

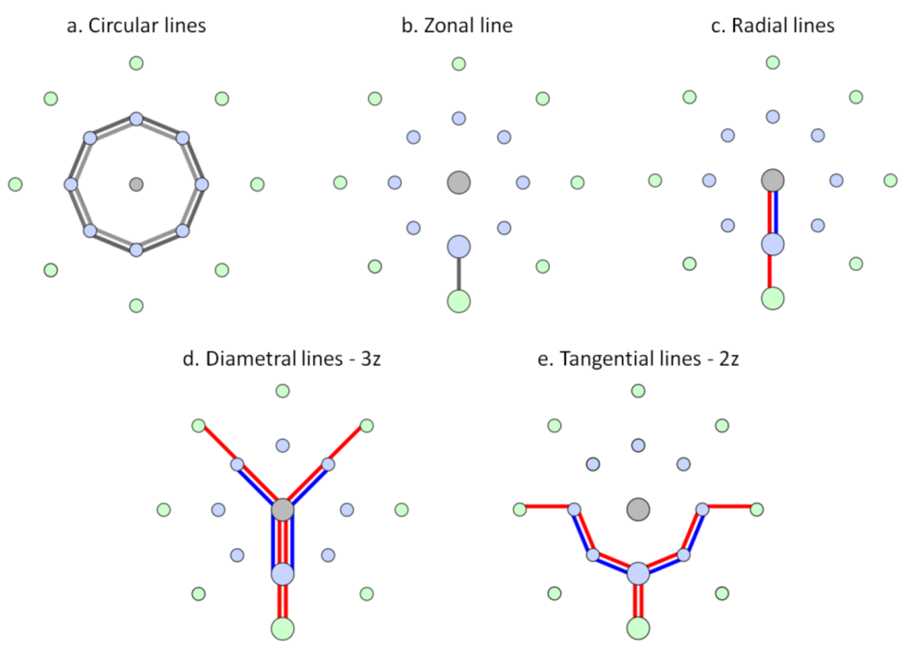


**Figure 2. Basic lines representation**

Line integration creates networks that can be classified according to their strategy, and described easily following the presented typology. For example, in a feeder-trunk structure the trunk lines are arranged in corridors such that most users need to transfer to reach their destinations; considering a city with eight zones, this structure includes eight zonal lines (feeders), four short diametric-4z, and eight short diametric-3z lines (acting as trunks), complemented with two circular lines.

### 2.2 Problem formulation and solution approach.

Finding the best lines-structure is a complex problem that justifies its resolution through heuristics in real city networks. However, when using the PCM with a given set of parameters that defines a city, one possible approach is to consider a reasonable set of pre-defined lines structures and then find for each structure $s$ the optimal frequencies and vehicle sizes for every line that forms the structure such that the total value of the resources consumed ($VRC_s$) is minimized. $VRC_s$includes operators' costs ($C_{op}$), infrastructure costs ($C_{in}$), and users' costs ($C_{us}$), which is their equivalent time spent (access, waiting, in-vehicle, and transfer). Once the minimum $VRC_s$has been found for all structures in the pre-defined set, the structure with the minimum optimized$VRC$is the best (within that set).

As evident, what we have called "a reasonable set of lines structures" on one hand does not exhaust all possible cases in the PCM; on the other hand, identifying all structures when two technologies are considered is unfeasible even for a small number of zones. To have an idea, the design analysis in one corridor (which can be regarded as one zone in the parametric city model) involves 13 structures if only one technology is considered, but it jumps to 74 possible lines structures with two technologies (Jara-Diaz and Muñoz-Paulsen, 2021). In those cases, involving three or more zones, the number of bimodal lines structures grows substantially, such that enumeration is practically unfeasible even if the analyst has to choose only among generic lines structures as feeder-trunk, hub-and-spoke, direct, or exclusive. This difficulty makes it necessary to introduce a methodology to identify a limited number of reasonable lines structures to approach the design problem involving two modes when a city is represented with the PCM.

We propose to solve the transit network design with a four-step procedure as shown in Figure 3. The procedure starts with the *proposition of an initial network design*, involving lines defined by its itinerary, stops, and vehicle technology. The second step consists of the optimization of the proposed network in terms of frequencies and vehicle sizes of each of the lines involved; this generates the *resulting network design* where some lines may exhibit null or very low frequencies and/or load factors ($\lambda$) in some line sections. The ad-hoc elimination of these lines or line sections due to these conditions generates the *proposed network redesign*, which requires the calculation of new frequencies and vehicle sizes. This leads to a *resulting network redesign*, whose new operational variables might impact the cost function and its components, which requires verification and, if reasonable, a new (re)design. As evident, different initial designs could lead to different resulting networks. The steps where the operational design variables are optimized involve finding optimal frequencies and vehicle sizes for the proposed lines structure design. This requires the assignment of passengers to routes to reach their destination; the routes followed by the passengers, however, are not known a priori as they depend on the optimal frequencies, which in turn depend on the demand pattern. This will be solved with an iterative procedure until convergence (in frequencies) is reached, assuming that the perception of users is deterministic and all their routes have the same fare. The procedure is described in Appendix A.

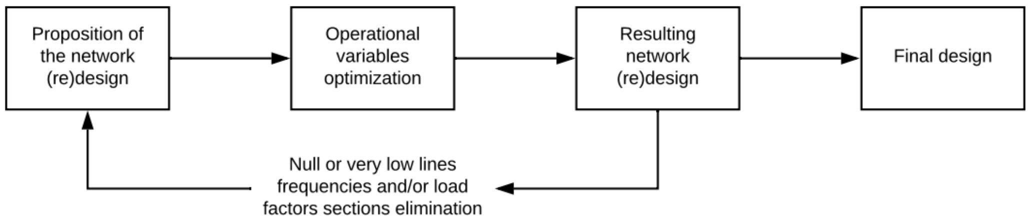


**Figure 3. Design procedure**

How to choose a starting or initial design in a specific city represented with PCM? One possible approach is to start with a sufficiently general structure involving both technologies, i.e. no transfers for each one. This would very likely generate a number of lines with null frequency such that the resulting design would constitute an interesting intermediate outcome by itself. An alternative approach is to use the observed lines structure to verify whether there are potential improvements. Yet a third approach is to start with the network of the technology that requires infrastructure - the "heavy" technology - as a base for the design, complemented with a (sub)-system of no-transfers services of light technology, incorporating all types of basic regular lines previously identified in section 2.1 as a sufficiently general case. Heavy technology infrastructure could be considered as exclusive (used for selected lines) or shared.

## 3. APPLICATIONS

The parameters are based on information from Santiago, Chile. The city is represented with a CBD and eight zones arranged with radial symmetry, each one containing a periphery and a subcenter. The demand pattern represents the morning peak with total patronage $Y = 300{,}000$pax/h, with 78% originated at peripheries ($a = 0.78$), and attraction parameters $\alpha = 0.25$ and $\beta = 0.22$, showing a slightly dispersed city. We consider two technologies: conventional buses running on exclusive lanes, and the subway. The formers run with intermediate speed and some irregular arrivals; the subway runs with high speed and regular arrivals. See Appendix B for a summary of demand and geometrical parameters, values of time, and modal data, all based on information from Santiago, Chile.

As described earlier, the initial design can be very general. The idea is to superimpose two complete systems of no-transfers services in both light and heavy technology (where services share the infrastructure). Each no-transfers system contains all types of basic regular lines introduced such that the total number of lines is 82 for each technology: two circular, eight diametric-4z (long and short), 16 diametric-3z (long and short), 16 tangential-2z (long and short), 16 tangential-1z (long and short), 16 radial (long and short), and 8 zonal lines. When optimizing such a general proposed network in terms of frequencies and vehicle sizes of each of the lines involved, many lines will vanish (null or very low frequencies) and some could present very low load factors $\lambda$ in one or more sections, leading to a much less dense bi-modal transit network.

Figure 4 shows the resulting design (no further iteration was needed, with all $\lambda$>0.5); only the lines (in one-direction) departing from one zone and the two circular lines are shown. Load factors in each line section are included in brackets. The loads in all lines in the P-SC sections are equal or close to one; in the reverse direction, however the demand pattern described earlier makes the load in all SC-P sections nil (long diametric and long tangential lines). The first two columns of Table 1 summarize the results including frequencies and vehicle sizes of each line for both technologies, and general system indicators.

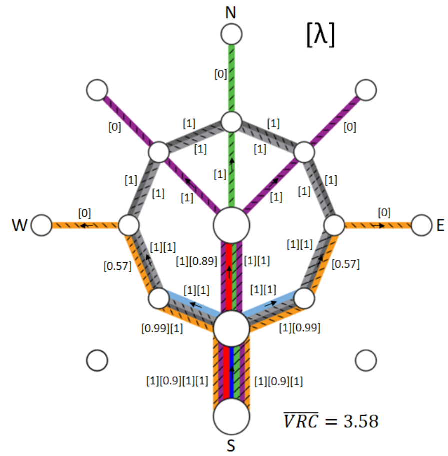


**Figure 4. Resulting design from an unconstrained initial design**

The second interesting initial design consists of the observed lines structures (both technologies) to explore whether it can be improved. The prevailing subway network in 2016[1] can be represented as having four diametric-4z lines reaching the subcenters (Villalobos, 2018), each with exclusive infrastructure. The bus network is represented by the long diametric 3z and 4z lines, but complemented by the circular lines. The resulting design (Figure 5b) included arcs with very low load factors (1%) that were eliminated for the proposed redesign (Figure 5c), keeping those with a load factor larger than 30%. The resulting redesign (Figure 5d) kept the proposed lines structure with somewhat different frequencies and vehicle sizes regarding the first resulting design; the final design is summarized in columns 3 and 4 of Table 1.

Finally, let us examine a slightly more flexible alternative approach, which is to consider a stylized representation of the observed heavy technology network (defined in the second approach) complemented with the complete system of no-transfers bus services (defined in the first approach). This proposed network is shown in Figure 6a (for one origin zone only). The resulting design involved load factors $\lambda$>0.35 so it was kept as the final design, presented in Figure 6b. As expected, the resulting VRC is larger than the VRC obtained with the most flexible one - the first (benchmark) case - but lower than the previous case where the initial bus network was less flexible. Detailed results are summarized in columns 5 and 6 in Table 1.

The synthesis in Table 1 confirms that the most general initial design yields the lowest VRC by means of large load factors in at least one direction in every arc. This benchmark solution has nearly no transfers, contains a large number of subway lines including two (very frequent) circular ones, and medium to small sized buses. The most constrained initial design (somewhat mimicking the observed structure) yields a solution with a large number of transfers (close to what was planned in Transantiago) and the largest VRC. The third initial design is quite interesting because the flexibility introduced in the bus subsystem makes the VRC decrease, but

[1] At that date there was 104 km of track; differences in length with the diametric lines are due to density at the CBD and one line that reached a periphery. Since that year, 40 km of subway track have been added.

without reaching the level of the general approach. Note that the best design solution reached with the most flexible initial design involves larger operational and infrastructure subway costs that are superseded by the reduction in users' costs, particularly in-vehicle time and transfers. It is particularly relevant to note that the (inferior) solution with the third approach for initial design can be improved by expanding the subway (as observed in recent years) and rearranging the bus lines.

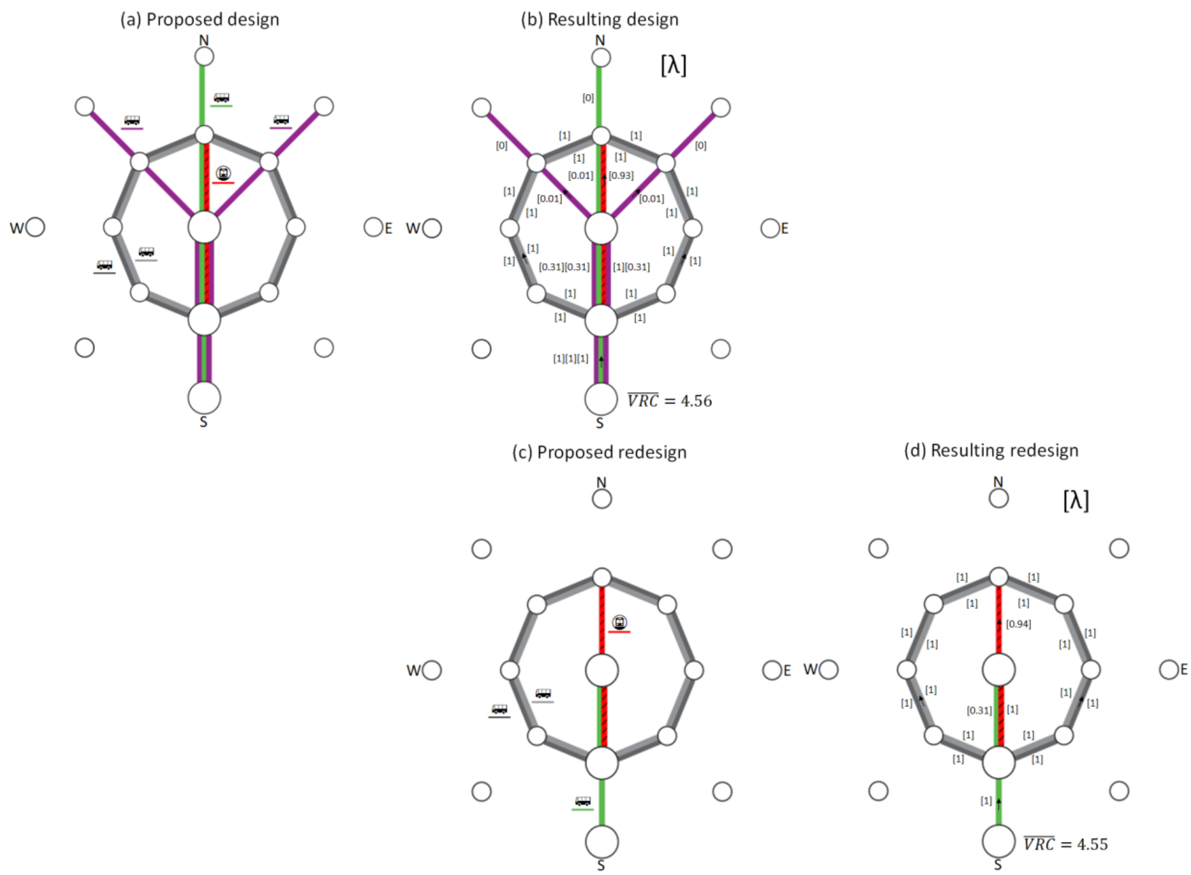


**Figure 5. Observed lines structure as proposed design**

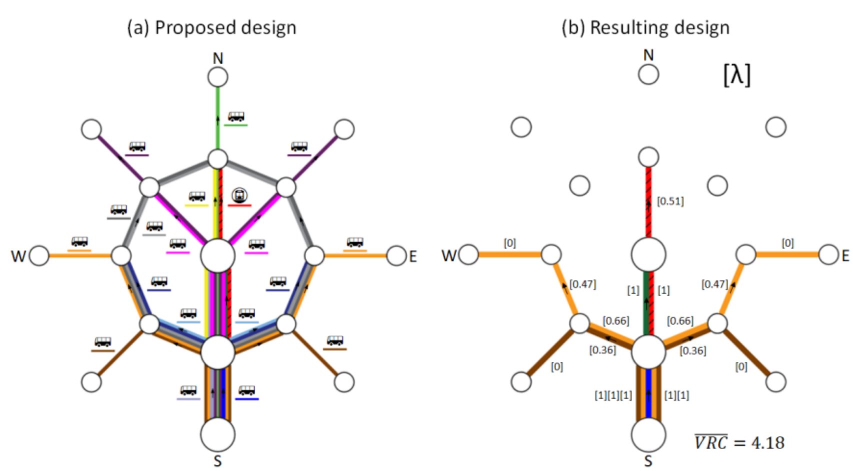

**Figure 6. Initial (observed heavy technology) and resulting designs**

**Table 1. Aggregate description of the final designs in Figures 4, 5 and 6.**

| | General lines structure | | Observed lines structure | | Observed heavy technology lines | |
|---|---|---|---|---|---|---|
| | Bus | Subway | Bus | Subway | Bus | Subway |
| Number of lines | 24 | 22 | 10 | 4 | 32 | 4 |
| Total lines length (km) | 554.5 | 1527.4 | 418.5 | 160.0 | 1207.4 | 160.0 |
| Fleet size (veh) | 1641 | 409 | 6798 | 196 | 5025 | 196 |
| Hourly frequency and vehicle size | | | | | | |
| Long diametral-4z | 0 | 9.5 [690] | - | - | 0 | - |
| Long diametral-3z | 0 | 8.0 [670] | - | - | 0 | - |
| Long tangential-2z | 0 | 7.2 [529] | - | - | 14.8 [117] | - |
| Long tangential-1z | 0 | 0 | - | - | 9.9 [79] | - |
| Short diametral-4z | 0 | 0 | - | 40.0 [366] | 0 | 40.0 [458] |
| Short tangential-1z | 9.0 [9] | 0 | - | - | 0 | - |
| Long radial | 5.7 [40] | 0 | 72.7 [101] | - | 0 | - |
| Short radial | 0 | 0 | - | - | 10.0 [27] | - |
| Zonal | 28.8 [40] | 0 | - | - | 40.6 [57] | - |
| Circular | 0 | 25.4 [187] | 34.0 [32] | - | 0 | - |
| Operators' costs ($us/h-pax) | 0.11 | 0.22 | 0.80 | 0.08 | 0.56 | 0.10 |
| Subway track length (km) | 209.2 | | 80.0 | | 80.0 | |
| Infrastructure costs ($us/h-pax) | 0.65 | | 0.25 | | 0.25 | |
| Users' costs ($us/h-pax) | 2.60 | | 3.42 | | 3.27 | |
| Access time (min) | 6.26 | | 2.19 | | 2.26 | |
| Waiting time (min) | 2.30 | | 1.23 | | 1.54 | |
| In-vehicle time (min) | 32.36 | | 54.61 | | 53.14 | |
| Transfers per trip | 0.07 | | 0.7 | | 0.54 | |
| VRC ($us/h-pax) | 3.58 | | 4.55 | | 4.18 | |

## 4. SYNTHESIS, CONCLUSIONS AND FURTHER RESEARCH

We have used the parametric city model (PCM) to propose and apply a methodology to design a strategic transit structure of public transport lines using up to two technologies. This methodology is based upon the proposal of a strategic initial design and successive redesigns based upon the frequencies and passenger loads of each line and segment involved. The applications using Santiago like parameters for the PCM and modes considered three initial proposed designs, one based upon two general very dense superimposed networks, one for each technology, while the two others began with an initial representation of the observed networks, one that considered the two technologies (bus and subway) and the other with the heavy technology only with the light one as dense as possible. The best result was obtained with the most general dense network that in only one step reached a bimodal lines structure where most of the original lines reached a nil frequency. The second best was obtained with an initial design that imposed the observed subway network only.

The main methodological conclusion is that the most general initial design seems to be the best starting point to converge into a good strategic single or bi-modal lines structure; as the resulting design could be unfeasible when analyzing a specific case with a pre-existing transit network, it can be considered as a very good benchmark, a reference to improve designs based upon observed transit networks. The applications show this in a very neat form: letting the observed heavy technology layout as mandatory, the light or flexible technology can be adapted to reach a better state, and/or the subway infrastructure can grow guided by the benchmark design. In our applications with Santiago-like parameters for demand and technologies it is quite interesting to verify that the benchmark can actually be achieved from the prevailing subway network by going into a scheme of lines where users do not have to transfer that much.

The last observation helps make a new research question. The procedure proposed and applied in this paper is based upon the **reduction** of the proposed design by deleting lines that present null or very low frequency and/or load factors in both directions. How to improve a design by letting the network **grow** in a smart direction? One approach emerges clearly from the experience presented here: start with a very dense general design superimposing both technologies in order to create a benchmark as guidance to grow. A second approach is to look at the passengers' routes in the final design in order to get an idea of the trade-off between increasing directness and adding new line segments**.** This should make new heuristics emerge.

## ACKNOWLEDGEMENTS

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**APPENDIX A. Finding frequencies and vehicle sizes for a given lines structure in the PCM.**

Problem (1) below solve the operational design problem, finding the optimal frequency $f_i^{(j)}$ and vehicle size $K_i^{(j)}$ of each line $i \in I$ (of technology $j$) that composes a specific lines structure. We define $\delta_{ij} = 1$ when the line □ is of technology □ (0 if not). Operators' costs include those associated to operate $B_i^{(j)}$ vehicles per hour, assuming that the unit cost is linear in the vehicle size (Jansson 1980) with $c_0^j$ and $c_1^j$ as given parameters. Infrastructure costs involve development, maintenance, and general costs; these are assumed to be proportional to the length $X^j$ of the network of technology $j$ with $c_2^j$ as cost per unit length and time. Users' costs include average waiting time ($\bar{t}_w$), in-vehicle time ($\bar{t}_v$), and access time ($\bar{t}_a$) per trip, each one multiplied by the respective value of time $p_h$ of the activity $h = \{a, w, v\}$; the average transfers per trip $\bar{R}$ is penalized by $p_R$ the pure transfer penalty.

$$\min VRC = \sum_{i\in I} \sum_j \delta_{ij}\left(c_0^j + c_1^j K_i^{(j)}\right) f_i^{(j)}\, t_{c_i} + \sum_j c_2^j X^j + Y(p_w \bar{t}_w + p_v \bar{t}_v + p_a \bar{t}_a + p_R \bar{R}) \quad (1)$$

s.t.

$$k_i^{(j)} \leq K_i^{(j)} \leq K_{max}^j \qquad \forall i \in I, j \in J \qquad (1a)$$

$$\sum_{i\in I} \delta_{ie}^{(j)} f_i^{(j)} \leq f_{max}^j \qquad e \in E, j \in J \qquad (1b)$$

The constraints (1a) impose that vehicle size in each line has to be large enough to carry the maximum vehicle load $k_i^{(j)}$ (maximum flow/frequency), and it should be lower than the maximum size available for that technology. Constraints (1b) impose that the sum of frequencies of lines of the same technology operating in each link $e$ cannot be larger than a maximum value given by capacity of the infrastructure (ways and stops) and safety considerations. We define $\delta_{ie}^{(j)} = 1$ when link $e$ belongs to the itinerary of line $i$ (0 if not). The first part of constraints (1a) is always active, as operators' cost increases with $K_i^{(j)}$such that fleet sizes and users' costs components can be expressed in terms of frequencies, which are the decision variables.

For each line, the fleet size is related to frequency through ${B_i}^{(j)} = f_i^{(j)}\ {t_{c_i}}^{(j)}$, where ${t_{c_i}}^{(j)}$ is cycle time which includes time in motion (first term in equation 2), and standing time at stops (second term in equation 2). We assume that time in motion depends only on the length of links of the itinerary and the commercial speed $v^j$(no congestion between vehicles). Boarding and alighting process is assumed to be sequential, such that the time at stops is the product of the average time to board or alight per passenger ($t^j$) times the number of passengers that board and alight at each node $m$ of the itinerary. The latter depends on demand pattern and on passengers' route choice, and can be represented through matrices $Z(i,m)$for boardings and $V(i,m)$ for alightings.

$$ {t_{c_i}}^{(j)} = \frac{2L}{v^j} \sum_{e\in E} \left( \delta^{(j)}_{ie(SC-CBD)} + g\delta^{(j)}_{ie(P-SC)} + r_n\, \delta^{(j)}_{ie(SC-SC)} \right) + t^j \sum_m \big( Z(i,m) + V(i,m) \big) \tag{2} $$

Regarding users' costs, total waiting time is given by the sum of experienced waiting times at each stage $q \in Q_r$ of the trip, associated to each route $r \in R_w$ covering each OD-pair $w$. We consider that waiting time is a proportion $\theta^j$ of the headway (inverse of the sum of frequencies) between vehicles of the common lines that minimize the total expected transit time at each stage of the trip. If $\varepsilon_{irq} = 1$ when passengers on route $r$ use line $i$ at stage $q$, the total waiting time is:

$$ t_w = \sum_w \sum_{r\in R_w} y_{wr} \sum_{q=1}^{Q_r} \frac{\theta^j}{\sum_{i\in I} \varepsilon_{irq} f_i^{(j)}} \tag{3} $$

Total in-vehicle time at each stage $q \in Q_r$ of the trip has three components: time in motion, involving all links of the stage and speed of line employed; time spent in-vehicle waiting for other passengers to board and alight at possible intermediate nodes $m$; and own average time spent to alight at transfer or destination node $d_q$.

$$ t_v = \sum_w \sum_{r\in R_w} y_{wr} \sum_{q=1}^{Q_r} \sum_{i\in I} \left( \varepsilon_{irq} \left( \frac{L}{v^j} \left( \sum_{e\in E} \left( \delta^{(j)}_{ie(SC-CBD)} + g\delta^{(j)}_{ie(P-SC)} + r_n\, \delta^{(j)}_{ie(SC-SC)} \right) \right) + t^j \sum_m \big( Z(i,m) + V(i,m) \big) + \frac{1}{2} t^j V\big(i, d_q\big) \right) \right) \tag{4} $$

Access time is composed of two elements: the potential time required to walk to platforms at a different level ($t^j_{az}$), if necessary at some stations depending on technology, and the walking time to the line of mode $j$ at origin nodes ($t^j_{ao}$). We assume that all lines converge to a single point at possible transfer nodes and at final destinations. For each OD-pair $w$, we consider that users are homogeneously distributed over the origin node (actually a line) with width $P$. In the case when only one technology is available, each user walks $t^j_{ao} = \frac{P}{4D^j v_a}$ to the closest line with speed $v_a$ (Chang and Schonfeld, 1991). In the case when two technologies are available, total weighted travel time related to each technology ($\tau^j$) will generate indifference points over the node width according to their lines spacing; therefore, each user walks to the closest line depending on its relative position, as shown in Figure A1.

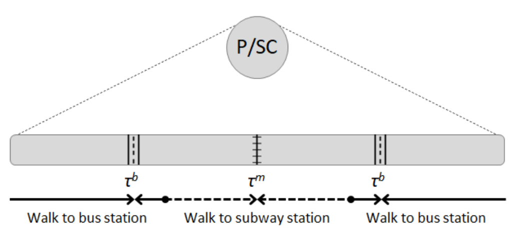


**Figure A1. Lateral walking at origin**

## APPENDIX B. Demand, geometrical and modal parameters, and values of time.

**Table B1. Generic parameters**

| Parameter | Meaning | Value |
|---|---|---|
| $Y$ | Total demand | 300,000 pax/h |
| $a$ | Proportion of trips that departs from peripheries | 0.78 |
| $\alpha$ | Proportion of trips departing from peripheries that goes to the CBD | 0.25 |
| $\beta$ | Proportion of trips departing from peripheries that goes to the own subcenter | 0.22 |
| $n$ | Number of zones | 8 |
| $L$ | Distance subcenter-CBD | 10 km |
| $g$ | Distance P-SC / Distance SC-CBD | 0.85 |
| $p_v$ | Value of in-vehicle time* | 2.74 $us/h |
| $p_w$ | Value of waiting time* | 5.48 $us/h |
| $p_a$ | Value of access time* | 8.22 $us/h |
| $p_R$ | Users' cost of a transfer (Pure Transfer Penalty, PTP) ** | 0.73 $us |
| $P$ | Origin node width | 2 km |
| $v_a$ | Walking speed | 4 km/h |

*From Ministerio de Desarrollo Social de Chile (2018).
**PTP=16 EIVM (equivalent in-vehicle minutes), a value that is within the range reported for multimodal networks (Garcia-Martinez et al. 2018).

**Table B2. Modal data**

| Parameter | Meaning | Bus | Subway |
|---|---|---|---|
| $c_o$ | Unitary cost per vehicle-time | 8.61 $us/h-veh | 80.91 $us/h-veh |
| $c_1$ | Unitary cost per seat-time | 0.30 $us/h-pax | 0.15 $us/h-pax |
| $c_2$ | Unitary infrastructure cost per length and time | 0.00 | 933.15 $us/h-km |
| $v$ | Running speed | 16 km/h | 40 km/h |
| $t$ | Boarding-alighting time per passenger | 2.5 s/pax | 2.5 s/pax |
| $f_{max}$ | Maximum frequency | 90 veh/h | 40 veh/h |
| $K_{max}$ | Maximum vehicle size | 160 pax/veh | 1440 pax/veh |
| $\theta$ | Arrivals regularity | 0.7 | 0.5 |
| $t_{az}$ | Access time to platforms | 0.00 | 1.00 min |
| $D$ | Lines density | 4 | 1 |

Operational costs parameters were estimated using information extracted from cost studies and annual reports (DTPM 2013, SECTRA 2015). Bus running speed parameter based upon Cubillos (2018).