ON APPLYING MIXED RP/SP MODELS TO POLICY FORECASTS: SOME NEW EVIDENCE

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ABSTRACT

The application of discrete choice models estimated at the individual level to forecast different transport strategic policies is common practice. However, as long as we move towards more complex demand models their specification as prediction tool is not immediate. This is also the case of relatively standard functions as the multinomial (MNL) and nested logit (NL) models, when estimated with mixed revealed preference (RP) and stated preference (SP) data, which are now also common practice. We found that their application in prediction brings to the fore some aspects that have been overlooked (i.e. not taken into account or applied unconsciously) or not fully understood.

The objective of this paper is to analyse in depth the problem of applying mixed RP-SP models in prediction, focusing on two aspects related to moving from the SP to the RP environment: (a) the problem of scaling specific SP parameters in prediction mode; we show that common practice may be incorrect in some situations; (b) the problem of defining consistent model structures across the RP and SP environments; we show that this does not have major consequences on model results even if the basic assumptions are not behaviourally correct.

Using several NL models with non-linear systematic utility functions estimated with mixed RP-SP data, we provide empirical evidence for the problems discussed from a theoretical point of view. Applying some strategies involving simple changes, we also estimate the errors that may occur when these models are not applied correctly.

1. INTRODUCTION

Since the early 90s when it was first proposed (Ben Akiva and Morikawa, 1990) the joint revealed preference (RP)-stated preference (SP) estimation method has become recommended practice. RP data, based on observations of actual choices and traditionally used in travel demand modelling, have many problems. In particular, when RP data are not measured with a high level of precision model structures and functional forms which would be appropriate with a fully disaggregate (i.e. properly measured) data set may not be selected leading to unknown bias in forecasting (Daly and Ortúzar, 1990). Conversely, SP data allow researchers to have good quality information (design under analyst's control) at a relatively small cost, since many observations can be obtained for each respondent. However, using SP data may mask a potentially large problem since good looking modelling results can be achieved with almost any SP survey, but if the technique is not used appropriately (for example using a non-customised design in a general context instead of focusing on specific behaviour), serious problems may remain undetected until forecasts are compared with actual outcomes (Ortúzar and Willumsen, 2001).

Thus the recommended approach involves using both data sources jointly, since it allows to exploit their advantages and overcome their limitations (Bradley and Daly, 1997; Louviere *et al*, 2000). The mixed RP-SP approach has now been used in many applications, both in research and/or in practical work, even with very complex structures. However, as it is often the case, much of the attention has been put into estimation leaving the correct use of these much improved models in prediction still in need of some aspects to be better understood.

The rest of the paper is organised as follows. Section 2 provides a brief review of the estimation problem when different data sources (i.e. RP and SP data in this case) are used. Section 3 analyses in depth the problem of using mixed RP-SP estimates for prediction, providing new evidence about some aspects that, to our knowledge, have not been fully explored. Section 4 gives a short description of the database used for the analysis and comments on the empirical results of the theoretical analyses discussed in section 3. In particular we examine the estimated models and analyse the effect on market shares of not using correct mixed RP-SP structures in prediction. Finally, our main conclusions summarised in section 5.

2. JOINT ESTIMATION FOR RP-SP DATA

As noted by Hensher (1994), using mixed RP-SP data to estimate choice models does not mean "*simply join the data*"; the scale factor in the indirect utility function must be considered. As the scale factor depends on the standard deviation of the error terms in the sample (for example in the MNL it is $\lambda = \pi / \sqrt{6}\sigma$), two identical models estimated with different data may give different estimated parameters, even if the individual choice process is the same. Given two sources of data, say one coming from a RP survey and the other from a SP one, the following random utility functions can be written:

(1)

where X_{RP} and X_{SP} are vectors of attributes common to both data sets (RP and SP) and β is the corresponding vector of parameters; Y and Z are vectors of attributes specific to each type of data[1], whose parameters are respectively α and γ. Finally ε and η are random terms associated to the RP and SP utilities respectively. Since the variance of the error term is associated to the data used to estimate the utility, σ^2_{RP} will be generally different from σ^2_{SP}.

An efficient and correct way to combine two different data sources (Ben-Akiva and Morikawa, 1990) is to scale one data set in order to achieve the same variance in both. It does not matter what utility is scaled, however commonly the SP utility is scaled:

$$\tilde{U}_{RP} = \phi U_{SP} \quad (2)$$

where to comply with the joint estimation requirement, the ϕ coefficient must be such that:

$$\phi^2 \sigma^2_{SP} = \sigma^2_{RP} \quad (3)$$

or, since $\sigma^2_{SP} = \frac{\pi^2}{6\lambda^2_{SP}}$ and $\sigma^2_{RP} = \frac{\pi^2}{6\lambda^2_{RP}}$, the scale factor becomes:

$$\phi = \frac{\lambda_{sp}}{\lambda_{RP}} \quad (4)$$

Therefore, the new ("scaled") utility function for the SP data set becomes:

$$\tilde{U}_{SP} = \phi U_{SP} = \underbrace{\phi \beta' X_{SP} + \phi \gamma' Z}_{\tilde{V}_{SP}} + \underbrace{\phi \varepsilon_{SP}}_{\tilde{\varepsilon}_{SP}} \qquad \phi \varepsilon_{SP} \sim (o, \sigma^2_{RP}) \quad (5)$$

and, the log-likelihood function for the joint estimation is:

$$L = \prod_{RP} \frac{e^{\lambda_{RP} V_{RP}}}{\sum_j e^{\lambda_{RP} V_{RP}(j)}} \cdot \prod_{SP} \frac{e^{\lambda_{RP} \tilde{V}_{SP}}}{\sum_j e^{\lambda_{RP} \tilde{V}_{SP}(j)}} \quad (6)$$

3. USING JOINT RP-SP ESTIMATION FOR PREDICTION

For prediction purposes, only the RP environment should be considered since it represents "real" behaviour. Thus, even if a joint RP-SP model is built in order to get better estimates, all the information must be moved to the RP environment when models are used in forecasting. This passage is not as easy as it can be imagined and some problems may arise, especially when more complex (but obviously correct) structures are used. In this section we analyse two of these problems: (1) scaling SP parameters by the RP-SP variance ratio when moved to the RP environment and (2) congruency of model structures across the RP and SP environments.

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[1] The specific attribute vector includes Alternative specific constants (ASC) and also generic variables (as times and cost) treated as specific for each subset of the data (i.e. with different parameters for the RP and SP data).

3.1. Scaling SP Parameters by the RP-SP Variances Ratio

As mentioned above, since only RP models can be used in forecasting, all information must be moved to the RP environment. The "common rule" is that scaling is required on those parameters moved from the SP to the RP environment (Hensher, 2002)[2]. However, since scaling is required because of the different nature of the data, the above statement is not generally true and could generate some errors. This is evident when interaction terms are included in the specification, as we usually estimate them with SP data only (because we can reduce correlation among attributes in a controlled experiment), but then we multiply the SP parameter by RP variables when the model is applied. Following the above rule we should scale the SP parameter, while actually in this case we do not. This is also the case when Alternative Specific Constant (ASC) are estimated across the RP and SP data sets; following the above rule we should always scale the SP parameter if we wanted to use it for prediction. However, in truth we should only scale the SP parameter if SP data are used for prediction; but not if RP data are used.

To demonstrate our point let us first consider, for example, the simple case in which a MNL is estimated for each source of data:

$$L_{RP} = \prod_{RP} \frac{\exp[\overbrace{(\lambda_{RP}\beta)}^{\beta_{RP}} X_{RP} + \overbrace{(\lambda_{RP}\alpha)}^{\alpha_{RP}} Y]}{\sum_j \%} \quad , \quad L_{SP} = \prod_{SP} \frac{\exp[\overbrace{(\lambda_{SP}\beta)}^{\beta_{SP}} X_{SP} + \overbrace{(\lambda_{SP}\gamma)}^{\gamma_{SP}} Z]}{\sum_j \%} \qquad (7)$$

the following parameters would be obtained:

$$\beta_{RP} = \lambda_{RP}\beta; \quad \beta_{SP} = \lambda_{SP}\beta; \quad \alpha_{RP} = \lambda_{RP}\alpha; \quad \gamma_{SP} = \lambda_{SP}\gamma$$

i.e., we would get different values for the same β parameters due to the unknown (inestimable) scale factors of the Gumbel distributions. When, instead, both source of data are estimated jointly and the SP utility is scaled as in equation (5), the log-likelihood function becomes:

$$L_{RP-SP} = \prod_{RP} \frac{\exp[\overbrace{(\lambda_{RP}\beta)}^{\beta'} X_{RP} + \overbrace{(\lambda_{RP}\alpha)}^{\alpha'} Y]}{\sum_j \%} \cdot \prod_{SP} \frac{\exp[\phi\overbrace{(\lambda_{RP}\beta)}^{\beta'} X_{SP} + \phi\overbrace{(\lambda_{RP}\gamma)}^{\gamma'} Z]}{\sum_j \%} \qquad (8)$$

and the following parameters would be obtained:

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[2] As suggested by one referee, a question arises whether it is correct to use RP probabilities including attributes not estimated with RP data, but only with mixed RP-SP data, in prediction. The problem arises because the scale parameter reflects the variance of the data and if SP information are moved into the RP domain, the RP variance should vary (i.e. the RP models used in estimation and prediction may have different variance). The problem certainly deserves a deeper analysis, however, it must be pointed out that in a joint RP-SP estimation the RP variance is affected (and it is usually reduced) by the inclusion of SP data in the estimation process; thus, in a joint estimation the RP variance (which is the overall RP-SP variance, being the SP variance scaled to be equal to the RP one) depends on the whole set of attributes, RP or SP specific. Therefore, in order to get consistency in the variance between estimation and prediction, the RP utility used in prediction should include all the attributes estimated in the RP-SP model, whether they are RP or SP specific.

$$\beta^{'} = \lambda_{RP}\beta; \quad \alpha^{'} = \lambda_{RP}\alpha; \quad \gamma^{'} = \lambda_{RP}\gamma; \quad \phi = \lambda_{SP}/\lambda_{RP}$$

i.e., we estimate all parameters scaled by the unknown (inestimable) RP factor scale of the Gumbel distribution. The key point is that when we multiply the whole SP utility by a scale factor (as in 5), we are effectively scaling the SP data in order to achieve the following equality[3]:

$$\phi X_{SP} = X_{RP} \tag{9}$$

So, if we could measure the Z attributes (i.e. those included only in SP data set) for the RP case, the following equality should also hold:

$$\phi Z = Z_{RP} \tag{10}$$

Finally, if we want to use the above results for prediction, the model probability should be:

$$P_{PR} = \frac{\exp[\overbrace{(\beta^{'})}^{(\lambda_{RP}\beta)} X_{RP} + \overbrace{(\alpha^{'})}^{(\lambda_{RP}\alpha)} Y + \phi \overbrace{(\gamma^{'})}^{(\lambda_{RP}\gamma)} Z]}{\sum_j \%} \tag{11}$$

where, correctly, the SP data (Z) are scaled by the ϕ parameter. However, if we apply the model with all attributes evaluated on RP data (i.e. even the Z attributes), what we should actually use is:

$$P_{PR} = \frac{\exp \overbrace{(\beta')}^{(\lambda_{RP}\beta)} X_{RP} + \overbrace{(\alpha')}^{(\lambda_{RP}\alpha)} Y + \overbrace{(\gamma')}^{(\lambda_{RP}\gamma)} \overbrace{(Z_{RP})}^{(\phi Z)}}{\sum_j \%} \tag{12}$$

so the $\gamma' Z_{RP}$ term should not be scaled by the ϕ factor, since we are using only RP data, the scale of which is consistent with the scale of the Logit model we are using for prediction. Considering again the example of the interaction terms, if we estimate the following mixed RP-SP model:

$$V_{RP} = \beta_{tv} tv_{RP} + \beta_c c_{RP} + ... \tag{13}$$

$$\tilde{V}_{SP} = \phi\left(\beta_{tv} tv_{SP} + \beta_c c_{SP} + \beta_{c*tv}\left(c_{SP} * tv_{SP}\right) + ...\right) \tag{14}$$

in prediction mode we have the following probability:

$$P_{PR} = \frac{\exp\left[\beta_{tv} tv_{RP} + \beta_c c_{RP} + \beta_{c*tv}(tv_{RP} + c_{RP})\right]}{\sum_j \%} \tag{15}$$

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[3] To get the same parameters (same scale) in the RP and SP data sets we multiply the whole SP random utility by a scale parameter. This is equivalent to data scaling in the sequential estimation method, where the equality is just a trick to get scale adjustment, not an assumption of equal units between both data sets.

because, even if the travel time and travel cost parameters β_t and β_c are generic for RP-SP data while β_{tc} applies only to SP data, all parameters are scaled by λ_{RP}. Therefore, since the variables *Tv* and *Cv* are the same (and in prediction we obviously use only the RP data), in this case we do not need to rescale the SP specific parameter.

Finally, the ASC deserve a note. Since ASC are not "data", in general we can say that they have to be always scaled when moved from the SP to the RP environment. However, the right way to move ASC depends on how they are specified and, differently from what is commonly believed, specifying ASC in RP-SP joint estimation is not a simple matter; it is certainly more complex than specifying the variables. An ample discussion on this subject can be found in Cherchi and Ortúzar (2003).

3.2. Congruency of Model Structures Across RP and SP Environments

The problem of congruency happens when, as it is often the case, different distributions of the error term are used for the RP and SP data in joint estimation. As an example we will analyse one particular case, but the analysis can obviously be extended to other cases. We have three alternatives either in the RP and in the SP data sets: two public transport options (bus and train) and the private car. However, as we will discuss in the next section, the SP experiment were based only on binary choice between two of the three modes, so nested correlation among sub-groups of alternatives (in our case between bus and train) could not occur.

Figure 1 shows the structure of our mixed model, where ϕ_1 is the structural parameter for the nested logit in the RP alternatives, and ϕ_2 is the scale parameter for the SP data in the joint estimation. Note that the two data sub-sets (i.e. RP and SP) are totally independent.

In prediction mode, the problem arises when we need to use SP specific variables, estimated under the hypothesis of absence of correlation, in the RP environment where correlation is allowed to exist between bus and train. In order solve the problem, two alternative structures were tested:

1. Scaling each SP data item without allowing for correlation among SP alternatives (Figure 2, case A).
2. Introducing correlation between bus and train in the SP data set to achieve consistency with the RP structure, even though we are estimating an SP structure which is different from that implicit in the experiment (Figure 2, case B).

The results will be discussed in the next section. However it is important to note that both structures are only ways to go round the problem but do not provide a clear answer. In fact, nesting is only a trick in joint estimation and nests are only behaviourally valid within a choice set (i.e. RP or SP). If one actually created a hierarchy within a choice setting, then one should use full scaling right through the levels (Swait, 2002).

Moreover, the solution is also strongly related to the specific case under study. In fact, if for example the SP alternatives include a new option which is just an improvement of the original one (train in our case, but substantially different since the characteristics are much improved), the structure we need to use in prediction depends on whether we believe that the unobserved attributes of the new alternative could be correlated with Bus in the same way as the existing option's unobserved attributes are correlated with Bus.

If it is postulated that the new option is different and replaces the original one, a simple MNL model with three uncorrelated alternatives could be used for prediction. However, in this case we would also be using parameters with different scale in prediction; i.e., estimated parameters deflected by a NL variance would be used in a MNL structure which imply a different scale.

4. MIXED RP-SP MODELS ESTIMATED AND DEMAND FORECAST

In this section we will analyse empirically the problems discussed in section 3. The data used for this analysis was collected in 1998 for a modal choice context involving two public transport modes (bus and train) and one private mode (car). To build the data bank a qualitative survey for gaining a good understanding of the phenomenon and two quantitative surveys (RP and SP) were carried out. In particular, in the RP case a 24-hour travel diary survey filled in personally by each respondent was used to collect data on current trips, as well as socio-economic characteristics; the sample size was 900 individuals.

The SP survey, conducted on a selected sub-sample (300 individuals) of the people who answered the RP questionnaire, had basically the objective of expanding the RP data bank and checking commuter responses to the introduction of a new train alternative (i.e. the current train service but with far superior characteristics). A choice experiment between the proposed new train service and the current transport mode was used. Moreover, an experimental design which allowed to estimate two-term interactions was used in order to account for non addictive effects of cost, frequency and travel time in the analysis. A final sample (i.e. mixed RP/SP data set) of 1,396 observations, composed of 338 RP individuals and 1,058 SP pseudo-individuals, was used for the model estimation. For more details see Cherchi and Ortúzar (2002).

Using these data, and the structure showed in Figure 1, several NL models were estimated with linear and non-linear utility functions including allowance for correlation among RP options using ALOGIT (Daly, 1998). The results, already discussed in depth in Cherchi and Ortúzar (2002), showed that interaction terms significantly improved model results, correlation between train and bus in the RP alternatives was highly significant, as well as the SP scale factor (model NL4 was judged our best model). The other model illustrated in Table 1 (NL5) was estimated using the structure in Figure 2 case B, as discussed in section 3.2 (we do not show the results of the structure illustrated in Figure 2 case A, since, as expected, they were equal to those obtained with the structure in Figure 1).

It is interesting to note that the introduction of correlation between bus and train in the SP alternatives does not have any effect on the model results. If we compare models NL4 and NL5, their parameters and t-test are almost identical, and the SP correlation is not significantly different from one. This is an expected good result since the SP experiment did not allow for correlation.

Even if not correct from a behavioural point of view, the fact that models NL4 and NL5 give the same results and especially that the interaction terms are almost equal, leave us less worried about moving interactions from the uncorrelated SP environment to the correlated RP one.

Using model NL4 (i.e. the best model estimated for the context under study), the variation in aggregate market shares for various simple policy measures were calculated and results compared both scaling (wrong approach) and not scaling (correct approach) the specific SP interaction parameters. The response to a change in prediction was calculated as the percent change in the aggregate share of mode *j* over the initial situation (do-nothing):

$$\Delta P_j = \frac{P_j - P_j^0}{P_j^0} \tag{16}$$

where P_j^0, P_j are the aggregate probabilities of choosing mode *j* before (do-nothing) and after introducing the measure, calculated by sample enumeration.

As can be seen in Figures 3 and 4, since the SP parameter in model NL4 is smaller than one (exactly 0.6268) if we scale the interaction SP parameters when included in the RP probability, we produce an overestimate of the alternative we are improving and, obviously an underestimate of the competitive options. Underestimation is obviously greater for the correlated alternatives. In particular, Figure 3 shows that for a reduction in travel time by Train if we erroneously scale the interactions the estimated percent change in the Train aggregate share (ΔP_j) is 37% bigger than if we do not scale the interactions. Figure 4 shows an analogous results for improving the car alternative. It is interesting to note that in this case the effect in the concurrent modes (bus and train) is much larger than in the previous case; this is because the effect depends obviously on the variables involved in the interactions and on the variables considered in the policy.

5. CONCLUSIONS

Joint RP-SP estimation has received a great deal of attention over the years and many major advances have been experienced both in theory and practice. Joint RP-SP estimation has also been used in many applications, including complex utility functions and large number of options. However, not many applications of mixed RP-SP model as prediction tools have been reported and, in particular, some important issues (about moving from estimation to prediction) do not appear to have been reported before.

In this paper we have tackled the problem of using mixed RP-SP models for prediction. In particular, we have analysed the problem of moving from the SP to the RP environments, as required when mixed RP-SP results are used for prediction. We have demonstrated that the "common way of doing" is not always correct, or at least is only correct in certain cases; we have also provided a general rule to apply RP-SP model in a prediction context. In this quest, we have also examined the problem of error structures, which is often different for RP and SP data; this should not be ignored when moving information from one source to the other. Unfortunately this problem has not a solution, but comparing the results from a correct structure (i.e. consistent between RP and SP) and an inconsistent one, we found not much difference (at least for the case of correlation among options) leaving us less worried about the common approach. However, we believe more evidence should be found on this respect.

Finally, applying some RP-SP models for prediction we analysed the effect of moving SP information into RP probabilities and compared the "common approach" with our new rule. We found that the potential errors in predicting demand for reasonably sensible policies can be quite high, thus raising an alarm about a problem which should be further examined.

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APPENDIX

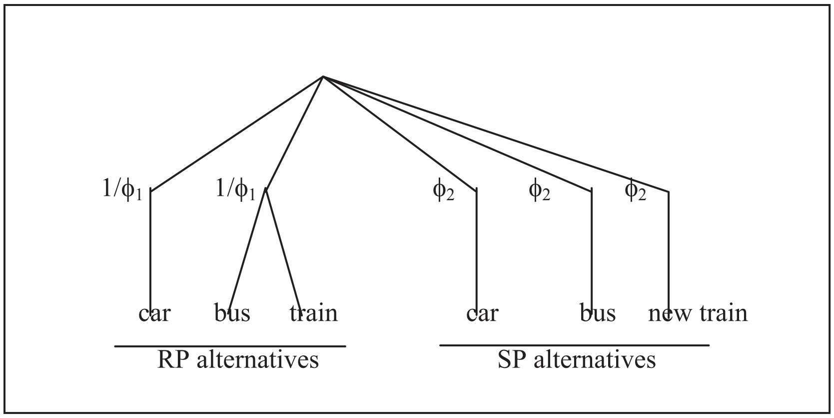

Figure 1: Structure Used to Estimate Mixed RP-SP Models

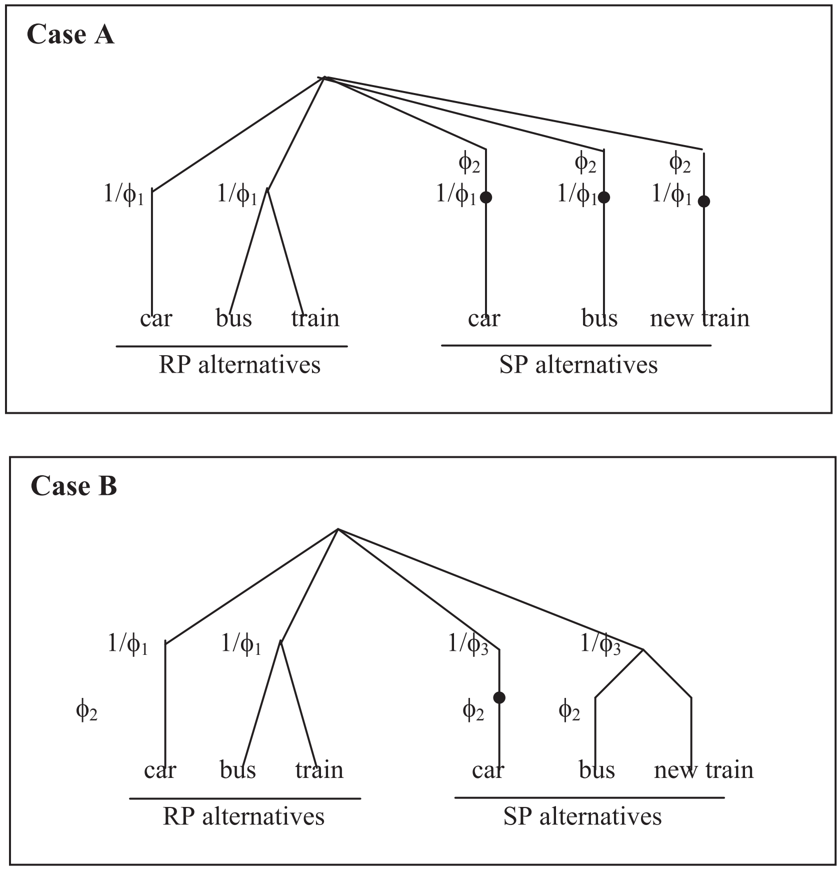

Figure 2: Alternative Structures Tested to Estimate Mixed RP-SP Models

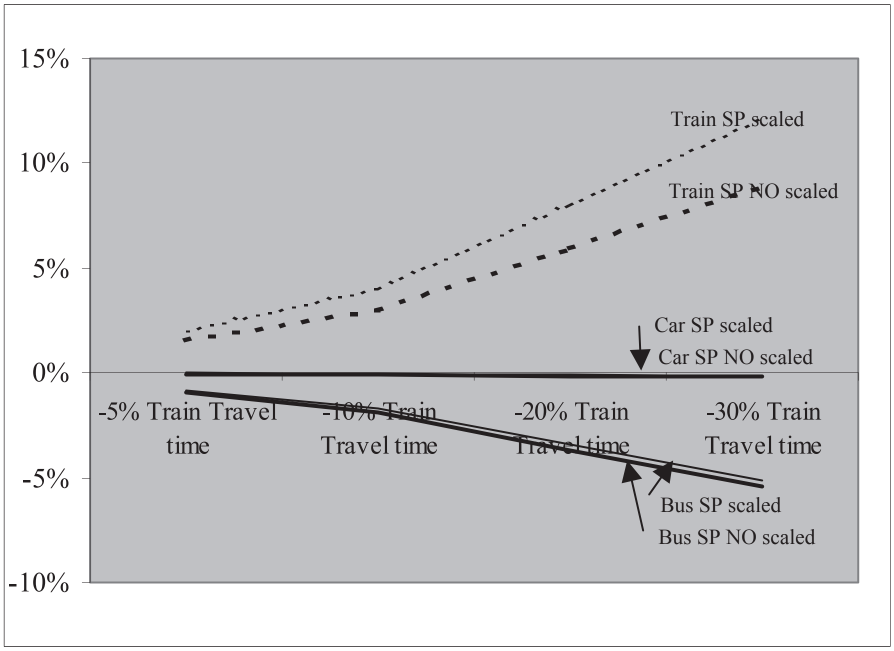

Figure 3: Effect of Scaling SP Parameters in Forecast Demand for Reduction on Train Travel Time

(N.B Bold lines refer to percent probability variation calculated not scaling SP interaction parameters when moved into RP domain)

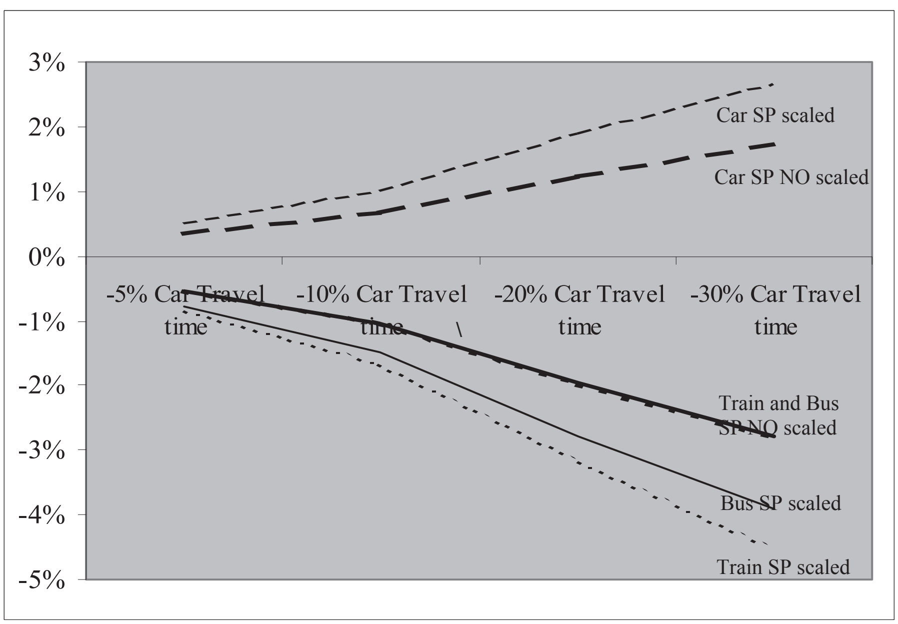

Figure 4: Effect of Scaling SP Parameters in Forecast Demand for Reduction on Car Travel Time

(N.B Bold lines refer to percent probability variation calculated not scaling SP interaction parameters when moved into RP domain)

Table 1
Model Estimation Results: Testing RP-SP Structures

Attributes	NL4 (Structures of Figure 1)	NL5 (Structures of Figure 2 case B)
Travel time PT	-0.03699 *(-1.6)*	-0.03786 *(-1.7)*
Travel time Car	-0.1753 *(-3.2)*	-0.1626 *(-3.2)*
Walking time	-0.06205 *(-2.6)*	-0.05973 *(-2.6)*
Cost/g	-0.02307 *(-2.7)*	-0.0225 *(-2.7)*
Frequency	0.5961 *(3.9)*	0.5900 *(3.9)*
Comfort 1	-3.189 *(-4.0)*	-3.235 *(-4.1)*
Comfort 2	-1.583 *(-3.7)*	-1.552 *(-3.7)*
Transfer	-1.131 *(-2.4)*	-1.013 *(-2.3)*
Early/Late (RP)	-0.2140 *(-2.6)*	-0.2209 *(-2.7)*
Car/Licences (RP)	10.14 *(3.4)*	9.537 *(3.3)*
*TravelTime*fare (SP)*	0.0009771 *(2.9)*	0.000838 *(2.8)*
*Travel Time*freq (SP)*	-0.01070 *(-3.2)*	-0.01048 *(-3.0)*
K_train (RP+SP)	-0.9780 *(-2.9)*	-1.001 *(-2.9)*
K_car (RP+SP)	1.369 *(1.6)*	1.200 *(1.5)*
ϕ_1 *(EMU)*[(1)] *(RP)*	0.4701 *(3.19)*	0.4989 *(2.83)*
ϕ_3 *(EMU)*[(1)] *(SP)*	--	1.266 *(1.11)*
ϕ_2 *(SP factor scale)* *[...]*[(1)]	0.6268 *(3.8)* *[2.28]*	0.5846 *(3.7)* *[2.66]*
L(max)	-743.450	-742.7194
L(C)	-995.077	-995.077
LR(C)	503.254	334.952
ρ^2*(C)*	0.2529	0.1536
Sample size	1,396	1,396

(1) t-test for the structural parameter for the PT nest with respect to one

(*) where not specified, attributes are constrained to be RP/SP generic