DELAYS AND QUEUES OF BUSES IN BUS STOPS

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RESUMEN

Existe profusa literatura sobre las relaciones entre demoras y longitudes de colas en función del grado de saturación en intersecciones. Sin embargo, poco se ha estudiado sobre relaciones similares en paraderos de buses. El objetivo de este trabajo es generar curvass demora y longitud de cola en paraderos para estudiar sus propiedades. Para esto, se aplicó un modelo de simulación de interacciones entre buses, pasajeros y tráfico en paraderos. Se encontró que las curvas de demora y cola difieren de las observadas en intersecciones. Además, no solo se debe considerar la capacidad nominal para el diseño y la gestión de un paradero, sino también el *grado de saturación*, la demora *dentro* de la parada y la longitud de la cola *aguas arriba* de la parada. Además, un semáforo *aguas abajo* de un paradero tiene importantes efectos en las demoras y colas, dependiendo del tiempo del ciclo y de la razón de verde efectivo.

Palabras claves: transporte público, paradas, demoras

ABSTRACT

There is much literature on relationships between delays and queue lengths as function of the degree of saturation at road junctions. However, little has been said about similar relationships at bus stops. The objective of this work is to generate delay and queue length curves in bus stops to study their properties. To that objective a simulation model of bus stop interactions between buses, passengers and nearby traffic was applied. It was found that delay and queue curves differ from that observe at road junctions. In addition, not only the nominal capacity should be considered for the design and management of bus stop, but also the *degree of saturation*, the delay *inside* the bus stop, and the queue length *upstream* the bus stop. Also, a traffic signal *downstream* a bus stop has important effects on delays and queues, depending on both, the cycle time, and the green time ratio.

Keywords: Public transport, bus stops, delays

1. INTRODUCTION

Traffic congestion is measured by the magnitude of the delay and queue length that take place in any road element, such as a section of a road, a road junction, an exit of a highway, a bus stops, etcetera. To obtain these indicators it is necessary to calculate the capacity and measure the flow that passes a point in a time period. The ratio between the traffic flow (q) and the capacity (Q) is called degree of saturation (x = q/Q), also called volume to capacity ratio (v/c), where v is the volume of traffic and c is the capacity (TRB, 2016). Henceforth we will talk about degree of saturation instead of volume to capacity ratio.

It is well-known in the traffic engineering community that there is a relationship between the degree of saturation, delays, and queues (Akçelik, 1998; Hurdle, 1984). Figure 1 shows a typical delay vs degree of saturation curve in a traffic signal. As can be seen, the delay rises gradually with the degree of saturation up to x = 0.7. From x > 0.7 there is a sharp increase in the delay. For that reason, a value of x = 0.9 is used for traffic signal timing in order to have reserve capacity due to random variations of the traffic flow. This threshold value is called “practical degree of saturation” (x_p).

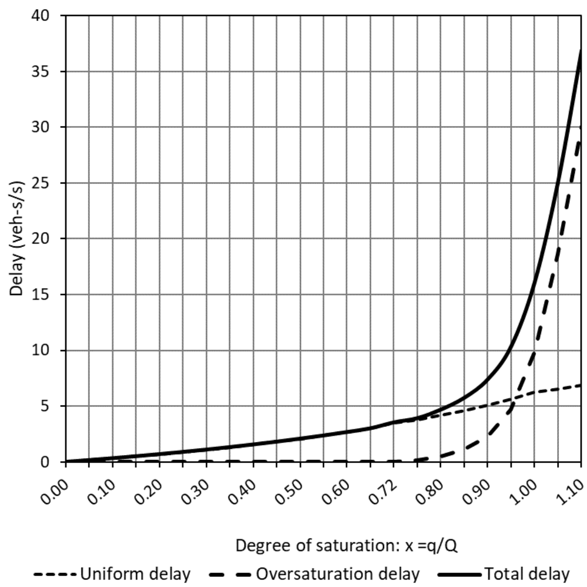

Figure 1. Delay rate as a function of the degree of saturation at a signalised junction.
(Cycle time = 100 s; green time ratio = 0.5; S = 1,800 veh/h per lane)

The model that explains the form of the curve in Figure 1 is shown in Equation (1), where all the variables are defined in APPENDIX 1. In the appendix the reader can also realize that d = D/q, where the units of the variables are [d] = [veh-s/veh] = [s], [D] = [veh-s/s] = [veh], and [q] = [veh/h].

$$D = \frac{qC(1-u)^2}{2(1-ux)} + N_0 x \qquad (1)$$

In this equation, the first term is the uniform delay, and the second term is called the random plus oversaturation delay. As can be seen, as x increases, the uniform delay increases because x is in the denominator. Similarly, the oversaturation delay increases more than proportional because x multiplies N_0 and, in turn, N_0 depends on x.

There is much literature on curves delay vs degree of saturation, d(x), and queue vs degree of saturation, L(x) at road junctions, however, little has been said about these relationships at bus stops. Therefore, the objective of this work is to produce d(x) and L(x) curves in bus stops by means of a microscopic simulation model called PASSION (Fernández, 2010), which allows a detailed analysis of the interactions between buses, passengers, and nearby traffic conditions in bus stops.

The d(x) and L(x) curves will be useful for studying policy operation as well as physical design of bus stop. For instance, the capacity of the bus stop, the number of berth (loading positions) of the stop area to accommodate the maximum number of buses that arrives, the dwell time of buses, the number of buses queueing upstream the bus stop if this has not enough capacity, and the effect on these outputs of a traffic signal located downstream the bus stop.

This article is divided in five chapters, including this introduction. Chapter 2 summarises the bibliographic review on modelling the delays and queues at road junctions and how the same approach can be applied to study these variables in bus stops. In Chapter 3 we explain the simulation methodology defining the capacities of the simulation model, the layout and operation conditions of the bus stop considered for this study, and two simulation scenarios: with and without a traffic signal downstream the bus stop. Chapter 4 show a working example which aims is to explain potential applications of this study. Finally, some comments on our work are presented in Chapter 5.

2. BIBLIOGRAPHIC REVIEW

A review of delays and queues at traffic signals can be found in Hurdle (1984) who cites previous works of Kimber and Daly (1986), Kimber and Hollis (1979), Akçelik (1988) and Akçelik (1981). The most important issue of these works is how to calculate delays and queues when flows approach to capacity.

The traditional queue theory states that in steady state if $x \rightarrow 1$, i.e., $q \rightarrow Q$, d (x) and L (x) tend to infinity. However, in the real word, no one has observer infinite delays or queues since they would require infinite time to form. The solution, for a finite time span, was provided by Kimber and Hollis (1979) defining a coordinate transformation function (Figure 2).

In the figure, L_t is the predicted queue length for any value of x at the end of a calculation period t; L_0 is the queue at the beginning of the calculation period; L_e is the steady state queue length predicted by the traditional queuing theory (Little, 1961); and L_d is the queue length when $x > 1$. The function in Figure 1 can also applied to delay calculation (Kimber and Daly, 1986). In fact, following Little (1961), the relationship between the queue length and delay is $d = L/q$.

In bus stops, however, a different behaviour has been found. First, the definition and calculation of capacity is different. In a junction the capacity is defined as the maximum number of vehicles that can cross the stop line in a given time period under prevailing, roadway, traffic, and control conditions (TRB, 2016). In other words, capacity can be understood as the maximum number of vehicles that can enter the junction in a given time period. At a signalised junction, the capacity is Q = uS, where the variables are defined in APPENDIX 1. The saturation flow S can be measured at the junction (RRL, 1963) or using well-known standard values (Akçelik, 1998). Therefore, the calculation of capacity at traffic signals only requires two variables: the signal timings, and the value of saturation flow.

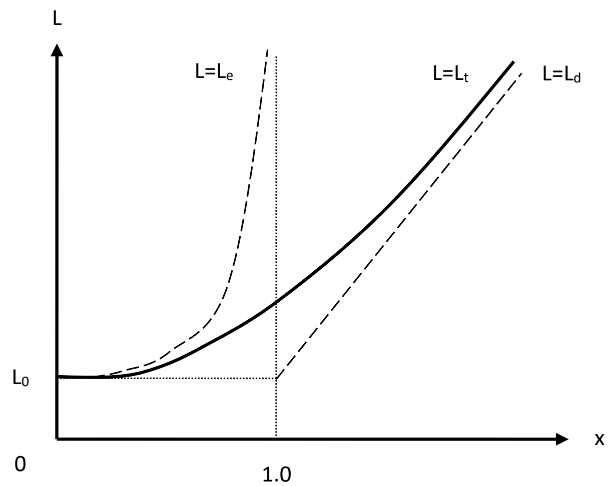

Figure 2. Coordinate transformation function to calculate queue length at junctions.

Following the analogy with the capacity of a junction, the capacity of a bus stop can be defined as the maximum number of buses that can enter the bus stop under prevailing conditions (Gibson et al, 1989), and it can be calculated as shown in Equation (2).

$$Q_B = \frac{3{,}600\ n}{\left(\frac{n}{S_b}\right) + t_d} \tag{2}$$

In Equation (2) Q_B is the capacity of a bus stops in buses per hour (bus/h); n is the average number of buses that can enter the bus stop; S_b is the saturation flow in the lane providing access to the bus stop (bus/s); t_d is the average time during which the bus stop is occupied for previous buses (s), and 3,600 is a unit conversion factor. When the bus stop has a single berth, n = 1 and t_d is the dwell time, i.e., the time for boarding and alighting passengers, as shown in Equation (4) below.

Another approach is that of the Transit Capacity and Quality of Service Manual (HCQSM) (TRB, 2013) and the Highway Capacity Manual (TRB, 2016), as shown in Equation (3).

$$Q_N = \frac{3{,}600(g/C)N_{eb}}{t_c + t_d(g/C) + Z_a c_v t_d}, \tag{3}$$

where Q_N is the capacity of a bus stop with N berths or loading points (bus/h); N_{eb} is the effective number of berths; (g/C) is the green ratio of the traffic signal downstream the bus top (if any), otherwise, (g/C) = 1; t_c is the clearance time of the bus stop (s); t_d is the average dwell time (s); c_v is the coefficient of variation of the dwell time; Z_a is the standard normal variate corresponding to the failure rate "a"; that is, the probability that one bus cannot enter the loading area because this is full of buses; and 3,600 is a unit conversion factor.

The value of Q_N depends on the average dwell time t_d which, in turn, depends on the number of boarding and aligning passengers and some parameters (Equation (4)).

$$t_d = P_a t_a + P_b t_b + t_{oc}, \qquad (4)$$

where P_b and P_a is the number of boarding and alighting passengers through the busiest door during the 15-min peak period, respectively; t_b and t_a is respectively the average boarding and alighting time per passenger (s/pass); t_{oc} is the door opening and closing time (s). For details about the parameters N_{eb}, t_c, t_a, t_b, Z_a, and c_v the reader can refer to the Highway Capacity Manual (HCM) TRB (2016). In the case that N = 1 berth, (g/C) = 1, and $c_v = 0$ (constant dwell times), Equation (3) is reduced to $Q_N = 3{,}600/(t_c + t_d)$. Note that in the case of the Gibson et al (1989) model $(1/S_b) = t_c$, so both models are the same.

Having defined the capacity of a bus stop the degree of saturation of a bus stop can be defined as $x_b = q_b/Q_b$, where q_b is the flow of buses arriving at the bus stop (bus/h) and Q_b is the capacity under some particular conditions, for example, random arrivals of buses and regular arrivals of passengers.

An application of Equation (3) is shown in TRB (2003) for planning purposes of bus facilities, such as the capacity of a bus lane in an arterial road. They developed graphs of bus stop capacities as a function of t_d and various cases of (g/C) for some values of Z_a, c_v. An example is shows in Figure 3 for a failure rate a = 10%, $c_v = 0.6$, and one-berth bus stop on the kerb lane. From the figure, the capacity of a downtown bus stop is about 76 buses per hour if (g/C) = 0.6 and $t_d = 15$ seconds. Assuming an average boarding time of 3 s/pass, this means 5 passengers boarding each bus. However, if $t_d = 40$ s, i.e., 13 passengers boarding each bus, the bus stop can cope with only 33 bus/h. This type of graph would allow public transport planners to know, for example, the critical bus stop on an arterial road.

Equation (3) is straightforward to perform these types of analyses, but it rests on empirical parameters coming from case studies in U.S. cities (Levinson, 1983; St. Jacques and Levinson, 1998); in particular, N_{eb}, t_c, and c_v. As a result, some analysis cannot be performed with this approach. For example, bus bunching or buses following a given headway distribution, e.g., Cowan (1975), passengers arriving in batches from a metro station or following some arrival function (e.g., Exponential). In such cases, the bus stop capacity may be quite different, and so the value of x_b and, in turn, $d(x_b)$ and $L(x_b)$. This has motivated the developing of a small number of simulation models to deal exclusively with bus stop operations.

As far as we know, only two simulation models to study in detail bus stop operations can be found in the literature (Gibson et al, 1989; Fernández, 2010). It should be noted that most commercial microscopic simulation software is devoted to representing general traffic, considering buses as "large cars" that block the traffic from time to time and their dwell times are random values taking from a given distribution (e.g., Normal) (Cortés et al, 2010). A discussion about this issue can be found in SMARTEST (1999).

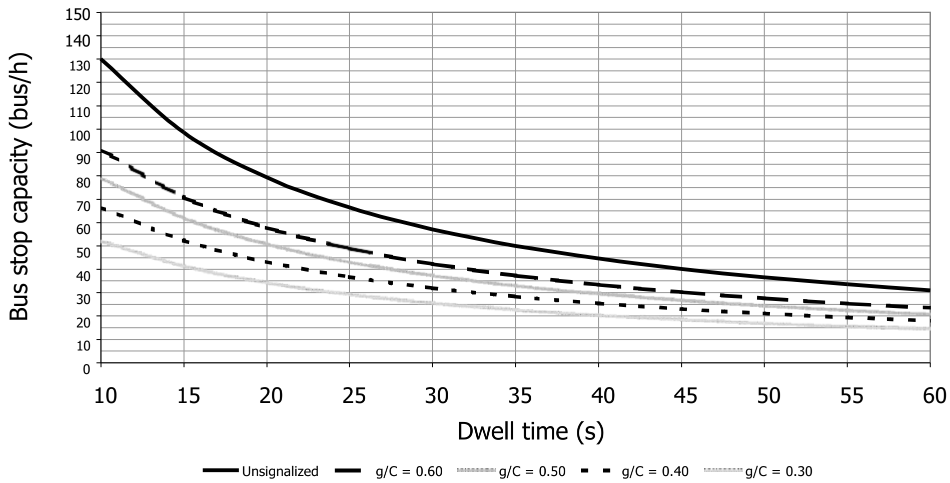

Figure 3. Bus stop capacity with on-line berth (TRB, 2003)

Gibson et al (1989) developed the simulation model IRENE to deal with isolated bus stops. The model assumes that buses arrive at the bus stop either at constant headways or following a shifted negative exponential distribution (Cowan, 1975). In addition, passengers arrive at a constant rate during the simulation period. The stop area may have one or more berths at which buses may follow either a First-In-First-Out (FIFO) discipline or they may exit from a berth overtaking a stopped bus, called OA (Overtaking-Allowed) discipline. Using this model, Gibson and Fernández (1995) obtained $d(x_b)$ and $L(x_b)$ curves for the cases shown in Table 1. Results are shown in Figure 4.

As can be seen from the figure, in all case $d(x_b)$ and $L(x_b)$ have a more gradual growth than at a road junction (Figure 1). According to Gibson and Fernández (1995) this behaviour is similar to that found in other transfer stations, such as ports. Also note that for a rather low degree of saturation (i.e., 0.7) the number of buses queueing upstream the bus stop is about one bus, which may be blocking the adjacent traffic lane. Our guess is that random batch arrivals play an important role in this respect. This was explored by repeating the experiment for Case 1, changing random arrivals for arrivals with constant intervals. The assumption was corroborated because the growth of the function moves to the vicinity of $x_b = 0.9$. Consequently, the form of arrivals is a decisive element for the estimation of delays and queues.

Table 1. Case description of Gibson and Fernández (1995)

Case	1	2	3	4
Boarding pass (pass/bus)	8.6	4.3	13.0	8.6
Alighting pass (pass/bus)	3.6	1.8	5.4	3.6
Exit discipline	FIFO	FIFO	FIFO	OA

In the same line, Tirachini (2014) by means of the model IRENE study the relationship between bus speed, stop spacing, number of berth and congestion at bus stops along a bus route. As a result, queuing delays at bus stops - the time spent by a bus to enter the bus stop - as a function of the bus flow, irrespective of the bus stop capacity where obtained.

Following the work of Gibson et al (1989), Fernández (2001 and 2010) developed a simulation model for studying bus stops interactions (PASSION: PAralell Stop SImulatiON). This is a microscopic simulation model of the interactions between buses and passengers in a bus stop. It also considers the traffic flow in the adjacent lane, and the effect of a downstream traffic signal. With this model curves $d(x_b)$ and $L(x_b)$ were obtained as explained in the next Chapter.

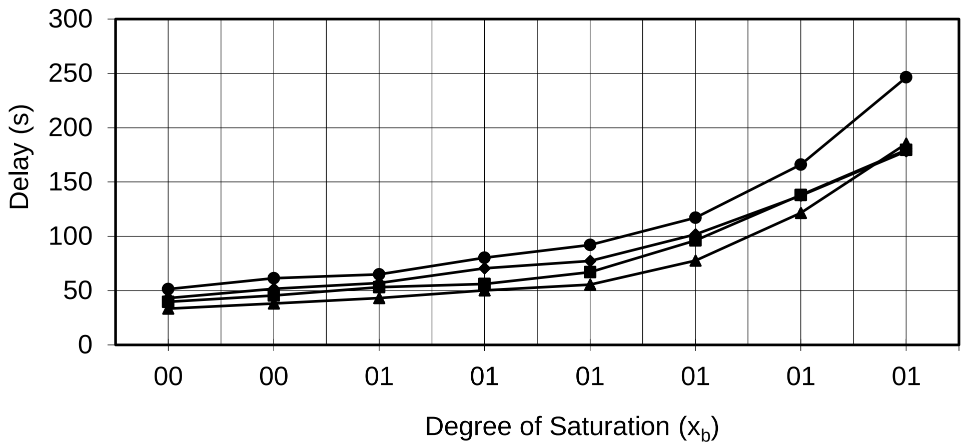

Figure 4. Delay at a 2-berth bus stop.

3. APPLICATION OF SIMULATION

3.1.The simulation model

We use the micro simulation model PASSION (Fernández, 2010) to calculate capacities, delays, and queues. In this model any arrival pattern of buses and passengers can be reproduced. Buses and passengers can arrive with constant headways, random headways following certain probability distribution defined by the user, scheduled arrivals, batch arrivals or actual arrivals, and combination of many bus routes with different frequencies and arrival patterns. The user may enter either an average boarding time per passenger or individual times for each passenger. This last option allows differentiating passengers by fare collection method (pre-payment, smart card, cash, etc.), type of bus (low floor, conventional with steps, width of doors, etc.), or physical condition of passengers (elderly, mobility impaired, healthy, etc.).

Once a bus finishes the boarding and alighting operations it leaves the berth according to four options. The first option is a free exit, that is, once the bus has completed the boarding and alighting operations, it can leave the berth without any further delay. The second option is the case when a traffic signal controls the exit from the bus stop. If the signal is red, extra delays are accumulated until the signal turns into green. A third option is when a bus has to re-enter the traffic stream from its berth, as in the case of a bus bay. In this case a gap-acceptance process is applied subject to the traffic flow in the adjacent lane. The fourth option represents any blockage ahead produced by other vehicles. Outputs of the model are the capacity, queues, and delays in the bus stop, as well as passenger waiting times and the number of waiting passengers on the platform.

The capacity of the bus stop and dwell times are computed as shown in Equation (5) and (6), respectively, with the same notation as in Equation (3) and (4), were the subscript i refers to each bus and j to each passenger boarding that bus. In Equation (5), N is the actual number of berths of the bus stop; n is the number of simulated buses; and t_{ei} is an extra time spent in the berth for any cause, e.g., the traffic signal in red, similar to the role of (g/C) in Equation (3). The term $Z_a c_v t_d$ is replaced by discrete simulation of passenger-bus interactions. More details about the model can be found in Fernández (2010).

$$Q_b = \frac{3{,}600\ N}{\frac{1}{n}\sum_{i=1}^{n}(t_c + t_{di} + t_{ei})} \quad (5)$$

$$t_{di} = \begin{cases} t_{oc} + \left(\sum_{j=1}^{P_{bi}} t_{bj} + t_{ai}P_{ai}\right), \text{if secuencial boading/alighting} \\ t_{oc} + \max\left(\sum_{j=1}^{P_{bi}} t_{bj}\ ;\ t_{ai}P_{ai}\right), \text{if paralell boarding/alighting} \end{cases} \quad (6)$$

In Equation (6) sequential boarding/alighting means that passengers first get off and then get in, as in metro trains. The parallel boarding/alighting means that passengers board through the front door and alight through the rear doors, as in the case of buses.

3.2.The simulation system

To obtain $d(x_b)$ and $L(x_b)$ curves with the simulation model, the system to be studied is shown in Figure 5. This is just an example of the layouts and operating conditions that the model can deal with. Other designs for which our simulation model can be applied are multiple-berth bus stop, skip bus stops, bus only lanes, etc.

The figure shows a one-berth bus stop located in the kerbside of the traffic lane upstream a traffic signal that, for any reason, may be on or off. The berth is 12-m long and 18 m upstream the bus stop are parking spaces (TRB, 1996). A bus arriving at the bus stop when this is occupied for the previous bus must stop upstream. Under certain circumstances this bus cannot stop close to the kerb and may bock the traffic. This situation is not an uncommon in the event of bus bunching or when the bus flow is high.

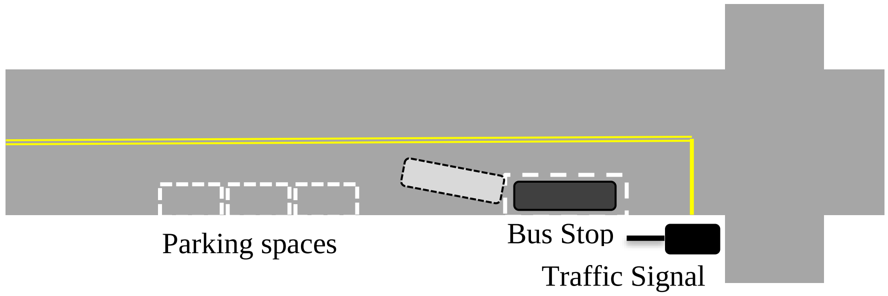

Figure 5. The system under investigation (not to scale)

a) Case 1: There is no traffic signal downstream the bus stop

Firstly, it is assumed that there is no traffic signal downstream the bus stop. For the simulation exercise the parameters are the following: t_{oc} = 5 s; t_b = 4 s/pass; t_a = 2 s/pass; t_e = 0. Also, exponential arrivals of buses and passengers is considered. In addition, buses have enough capacity (100 pass) so that waiting passengers can board the first bus that arrives. Therefore, bus overcrowding is not considered here for the sake of conciseness, but the model can deal with this situation. What the model *cannot* do is to simulate the dynamics of the passengers on board the bus.

The results of the function $Q_b(P_b)$ for 10,000 runs are shown in Table 2 and Figure 6. It can be seen from the table that the capacity of the bus stop for P_b = 10 pass/bus is 74 bus/h.

Table 2. Bus stop capacity as a function of boarding passengers.

P_b (pass/bus)	Q_b (bus/h)	P_b (pass/bus)	Q_b (bus/h)	P_b (pass/bus)	Q_b (bus/h)	P_b (pass/bus)	Q_b (bus/h)
1	196	6	102	11	69	16	52
2	166	7	94	12	65	17	50
3	144	8	86	13	61	18	48
4	127	9	80	14	58	19	46
5	113	10	74	15	55	20	44

The central line in Figure 6 is the average estimation of the capacity of one-berth bus stop. Note that the figure has the same shape that Figure 3 from TRB (2003). It should be noted that this is not intended to be a validation of our model, it is just a comment on the properties of the curves.

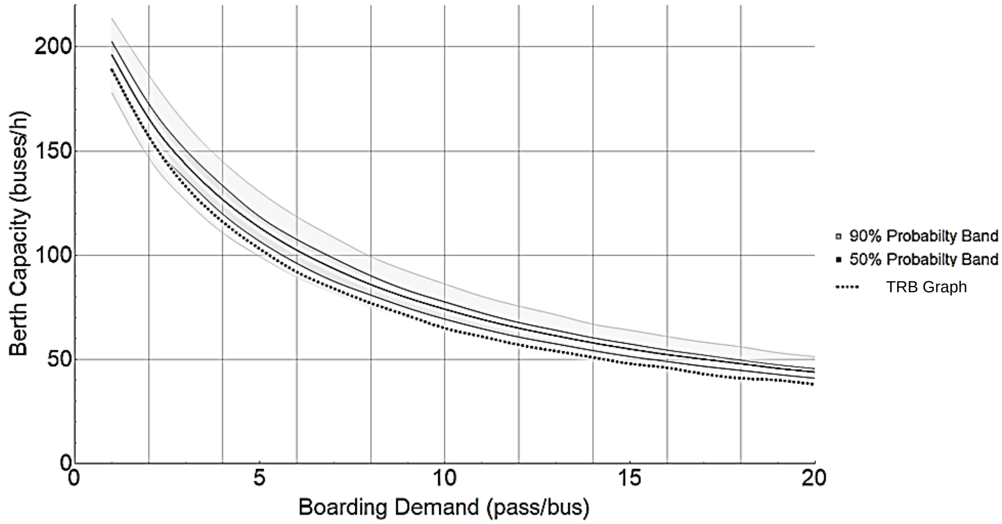

Figure 6. Capacity of a bus stop as a function of the number of boarding passengers

Table 3, Figure (7) and Figure (8) show the results of $d(x_b)$ and $L(x_b)$ curves obtained by Gibson and Fernández (1995). The reader can appreciate that the form of the $d(x_b)$ and $L(x_b)$ curves is similar (see Figure 4). It can also be seen from Table 3 that for $x_b = 1.0$, i.e., the bus stop is working at its nominal capacity, the queue length is 8.45 buses, and the delay is almost 4 min. These numbers are inacceptable for the operation of a bus stop. However, if the bus stops operate at 50% of its nominal capacity, i.e., $x_b = 0.5$, then $d(x_b) = 79$ s (1.32 min) and $L(x_b) = 0.39$ buses; that is, there will be one bus queuing almost 40% of the time, which is an acceptable operation.

Table 3. Delay and queue as a function of the degree of saturation.

Degree of saturation	Delay (s)	Queue length (bus)
0.1	55	0.02
0.2	61	0.05
0.3	67	0.10
0.4	72	0.21
0.5	79	0.39
0.6	88	0.67
0.7	97	1.15
0.8	112	2.01
0.9	134	3.69
1.0	177	8.45

The explanation of the different shapes of the curves can be explained as follow. According to Little (1961) the relationship between delay and queue length can be expressed as $L(x_b) = q \cdot d(x_b)$, where $L(q_b)$ is average queue length for a given x_b. As q_b can be write as $x_b \cdot Q_b$, then $L(x_b) = x_b \cdot d(x_b)/Q_b$. Therefore, $L(x_b)$ increases more than proportionally than $d(x_b)$ but divided by Q_b which is a large number. This is the reason for the shape of the curves: $d(x_b)$ increases smoothly, and $L(x_b)$ increases sharply but starting from a lower value.

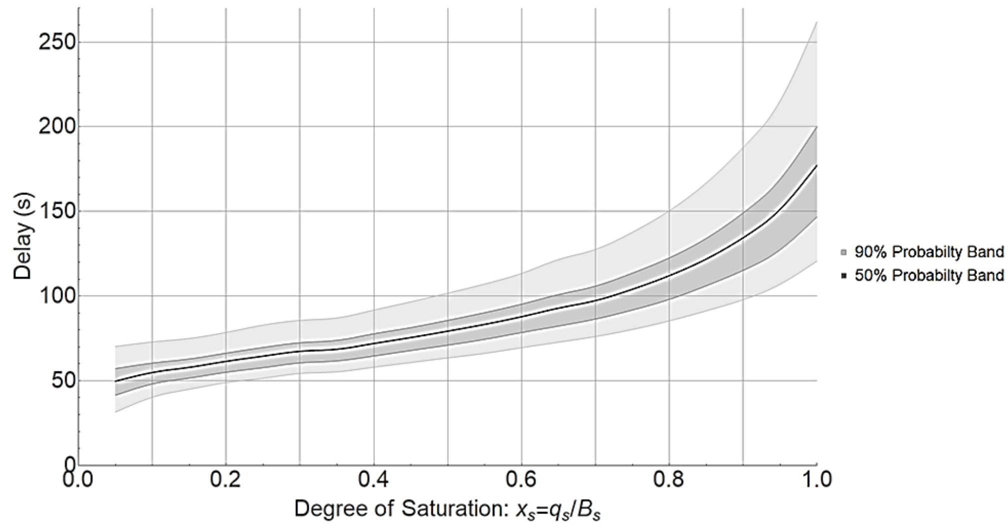

Figure 7. Delay in a bus stop as a function of the degree of saturation.

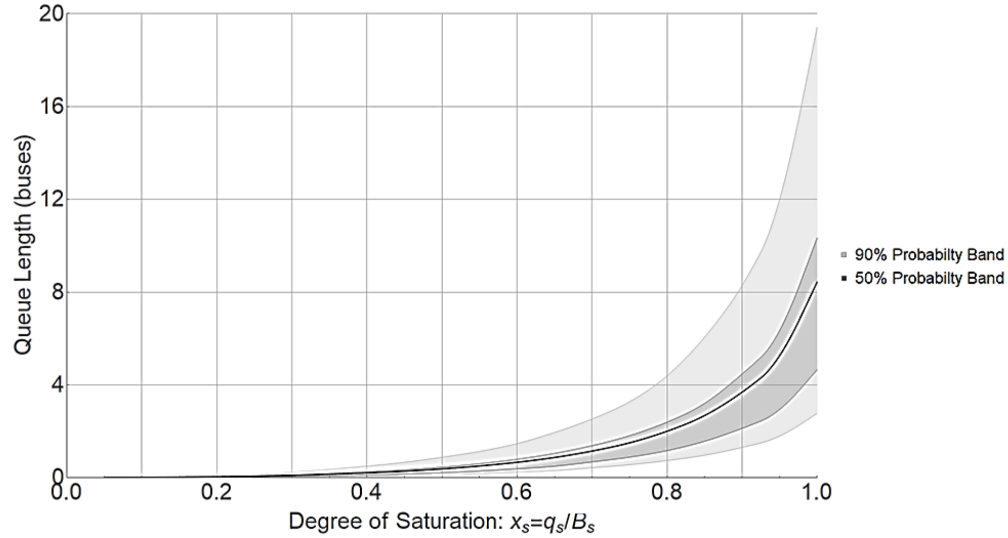

Figure 8. Queue length in a bus stop as a function of the degree of saturation.

b) Case 2: There is a traffic signal downstream the bus stop

Another important issue that can be studied using the simulation approach is the influence of a downstream traffic signal in the performance of a bus stop. In this case, $d(x_b)$ curves are generated for various cases of signal timing.

The TCQSM, mentions that factors affecting bus stop capacity are the cycle time, the location of the bus stop (near-side, mid-block, or far-side), and the green time ratio. But as show in Equation (2) only the green time ratio (g/C) is included in the calculation of Q_N. The other inputs are empirical parameters to consider the randomness in the arrivals of buses and passengers; that is, the coefficient of variation of dwell times (c_v) and Z_a for a given failure rate “a”. However, in a previous work Gibson (1996) fond that four variables have influence on the bus stop capacity: the cycle time (C), the green time ratio (g/C), and the physical distance from the traffic signal to the bus stop. He also mentions that in multiple-berth bus stops the exit discipline (FIFO or OA) also plays a role on the capacity.

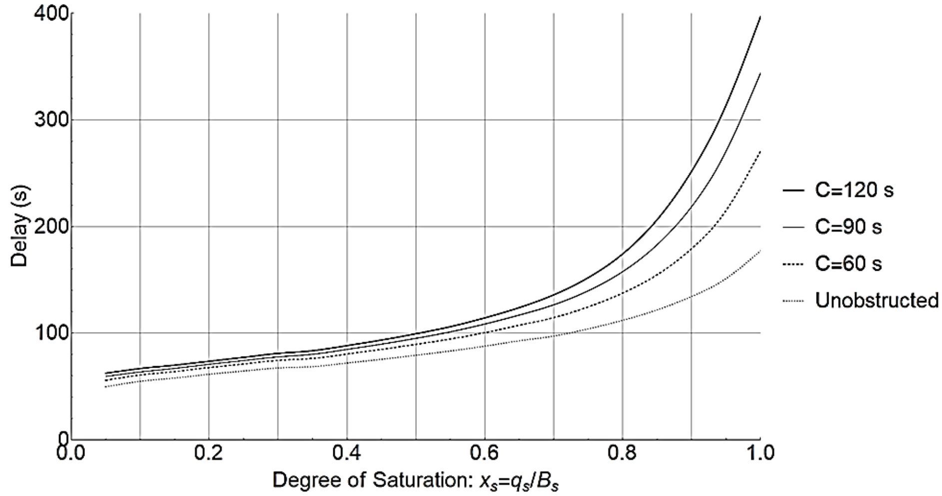

Figure 8. Delay as a function of the cycle time for (g/C) = 0.6.

In the following we show the effect of the signal timing on delay in bus stops. In order to carry out this simulation (g/C) = 0.6 is assumed, and three values of the cycle time are tested: 60, 90, and 120 s. Figure 8 shows the results.

In order to confirm this hypothesis, a number of simulations for a fixed 60-s cycle time and three values of (g/C) were performed. The results are shown in Figure 9.

According to the figure, for a given green time ratio, delays and queues are different due to changes in the cycle time. This means that not only (g/C) but also C has influence, as reported in Gibson (1996). As shown in the figure, for x_b = 0.6, as the cycle time increases, the delay also increases, and the worst case is for C = 120 s. In this case, the delay is 20% greater than for C = 60 s. However, in the case of queue length, the cycle time has no influence. Only when x_b is greater than 0.8, a difference of 20% is observed comparing C = 60 s and C = 120 s. But, if $x_b \geq 0.7$, the bus stop will be oversaturated because the queue length will be 4 buses or more. In conclusion, our findings indicate that to reduce delays and queues in a bus stop, the cycle time must be short (e.g., 60 s) and the green time ratio (g/C) greater than 0.6.

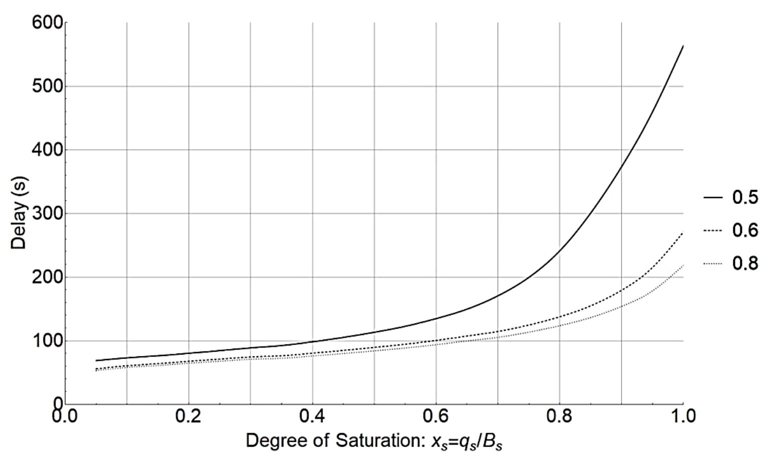

Figure 9. Delay as a function of (g/C) for C = 60 s.

It can be seen from the figure that delays and queues are lower as (g/C) increases. This is an expected result, just as it happens at signalised junctions. However, for (g/C) = 0.5 there is a marked increase of delays and queues compared with (g/C) = 0.6. In addition, the difference between (g/C) = 0.6 and (g/C) = 0.8 is negligible. Consequently, a green time ratio equal to 0.6 is sufficient to minimize queues and delays. As far as we know, this property of d(x) and L(x) curves have not been reported in the literature.

4. A WORKING EXAMPLE

As was demonstrated in the previous chapter, the calculation of $d(x_b)$ and $L(x_b)$ is useful for planning the operation of bus stops. An example of the use of the curves $Q_b(P_b)$, $d(x_b)$ and $L(x_b)$ is presented here. The objective of this example is to show how to manage delays and queues if the boarding demand of passengers is known. This demand is easy to know by counting the number of passengers waiting for the next bus that arrives.

Let us assume that in the critical bus stop of an arterial road the boarding demand is 15 pass/bus. The layout of that bus stop may be similar to that shown in Figure 5. The downstream traffic signal operates with 60-s cycle time and 0.6 green time ratio. Given the layout of the bus stop, the Traffic Authority state that no more than one bus can stop upstream the bus stop and the stopping time should be less than 2 min.

From Table 2 and Figure 6, the nominal capacity of the bus stop when 15 passengers are boarding is $Q_b = 55$ bus/h. To have only one bus queuing, both Figure 8 and Figure 9 show that the degree of saturation should be $x_b = 0.62$, almost irrespectively the cycle time. Note that if $x_b = 0.62$, the bus stop can only cope with 34 buses per hour (0.62·55). As a consequence, the headway (h) between buses should be less than 106 seconds (h = 3,600/34) or 1.76 minutes.

On the other hand, according to the same figures, to have a delay of 120 seconds the degree of saturation should be $x_b = 0.83$, which in this case strongly depend on the cycle time. Under this restrain the bus stop can serve 46 bus/h (0.83·55), i.e., one bus every 79 seconds or 1.31 min.

In conclusion, for managing the bus stop, given the restraints imposing by the Traffic Authority, our recommendation to the operator is that the maximum headway between buses must be about 1.8 minutes (the maximum between 1.31 min and 1.76 min).

As can be seen in this example, the utility of the curves $d(x_b)$ and $L(x_b)$ developed in this work is the application in the design and operation of bus stops. These curves would allow bus operators and traffic authorities to manage bus operations at the bus stops of an arterial road or along a bus route. From another viewpoint, the curves allow, given certain characteristics of bus operation, define the times of traffic signal to help manage the bus stops. Obviously, in the latter case, the traffic on the cross streets must be considered.

5. CONCLUSIONS

The objective of this work was to obtain $d(x_b)$ and $L(x_b)$ curves by means of simulation. This objective was achieved with the microscopic simulator of bus-passenger-signal interaction in a bus stop PASSION, making ten thousand (10,000) runs. Although the curves were obtained for particular inputs it was proven that their shape has general validity. Applications of these curves are planning facilities for buses, such as the identification of the critical stop, the need of another berth in a bus stop, the maximum number of buses per hour that can use a bus stop or a bus lane, among others.

Some properties of the curves were established. The most important is that the degree of saturation - the bus flow to bus stop capacity ratio - should be keep low (e.g., 0.5) for having few delays (e.g., less than 100 seconds) and the queue lengths should not exceed one bus during some percentage of the operation time of the bus stop. This means that not only the nominal capacity can be considered for the design and operation of bus stops, but also the degree of saturation, the delays, and the queue lengths.

It was be observed that the degree of saturation in bus stops is more restricted than in road junctions, where 0.9 is accepted. The reason is that in a bus stop $d(x_b)$ has a more gradual growth than in the case of junctions in which the curve is flat for low degrees of saturation ($x < 0.7$) and then grows rapidly. This is because at junctions the vehicles go through the stop line. On the contrary, in a bus stops the vehicles have to decelerate, stop to get in and off passengers and then accelerate, where the dwell time at busy bus stops may reach more than one minute.

It was also shown that traffic signals downstream bus stops have a significant effect on saturation and so in delays and queues. It has been recognised in the literature that the green time ratio has influence on the bus stop capacity. However, in this study has been found that the cycle time also plays a role. Our work has shown that the cycle time is more relevant in delays than in queue lengths. Although this also happens at traffic signals, it has not been mentioned in the literature in the case of bus stops.

The simulation experiments shown that $d(x_b)$ and $L(x_b)$ curves keep their properties under different conditions. These properties will allow engineers to deal with the problem of location, design, and operation of bus stops. For example, it may be possible to provide passive signal priorities for buses by shortening the cycle time and/or increasing the green time ratio. However, if due to network conditions a signalised junction must operate with a long cycle time and/or a low degree of saturation, the bus stop may be located at mid-block.

It should be mentioned that our results are limited to the simulation system and cases analysed in this work. They do not pretend to be the ultimate word on this topic. For instance, the parameters of our simulation exercise - clearance time, time for opening and closing doors, boarding and alighting times per passenger, and number of berths - were taken from typical values reported in the literature, as well as from our own field observations, but they do not represent some particular conditions. In that case new simulations considering the variables and parameters for that particular situation should be done. The way to carry out these other studies is described in this article.

The usefulness of the simulation models built to study the interactions between buses and passengers at bus stops could be established. It is true that commercial simulation models can also be applied for that purpose, but most microscopic simulation programs are dedicated to representing general traffic, considering buses as large cars that block traffic from time to time and their stop times are random values taken from a given probability distribution (for example, Normal) or from random tables.

Further research on this topic is the expansion of this approach to model a sequence of bus stops on an arterial road, in order to consider the impact of traffic congestion on public transport. Another aspect in which more research is required is the study of the effect on delays at bus stops of the density of passengers both inside vehicles and on platforms. This will allow considering the impact of overcrowding on board vehicles on the operation of buses. These works will be carried out as part of this line of research.

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APPENDIX 1. Notation and definitions.

q = flow of vehicles arriving at a traffic element (veh/h).

Q: capacity - maximum number of vehicles that can be accommodate at a traffic element - (veh/h).

x = q/Q: degree of saturation.

D: Average delay per unit time or delay rate (veh-s/s) or (veh)

d = D/q: average delay per vehicle (veh-s/veh) or (s)

L: queue length (veh).

C: cycle time of a traffic signal (s).

u = g/C: effective green ratio of a traffic signal.

$g = G - \lambda_1 + \lambda_2$: effective green time of a traffic signal (s).

G: displayed green time of a traffic signal (s).

λ_1: lost time at the start of the green time (s).

λ_2: gain time during the amber (s).

S: saturation flow - maximum rate of departure from a queue during the time period g - (veh/h).

N_0 = overflow queue - number of vehicles left queuing when signal change to red (veh).

$$N_0 = \frac{QT_f}{4}\left[(x-1) + \sqrt{(x-1)^2 + \frac{12(x-x_0)}{QT_f}}\right] \text{ if } x > x_0 = 0.67 + \frac{gS}{600} \qquad \text{(Akçelik, 1998)}$$

T_f: time period during which q and Q remain constant (h)

L_t: queue length end of a calculation period t.

L_0: queue length at the beginning of the calculation period t.

L_e: steady state queue length predicted by the traditional queuing theory when $x \leq 1$.

L_d: queue length when $x > 1$.